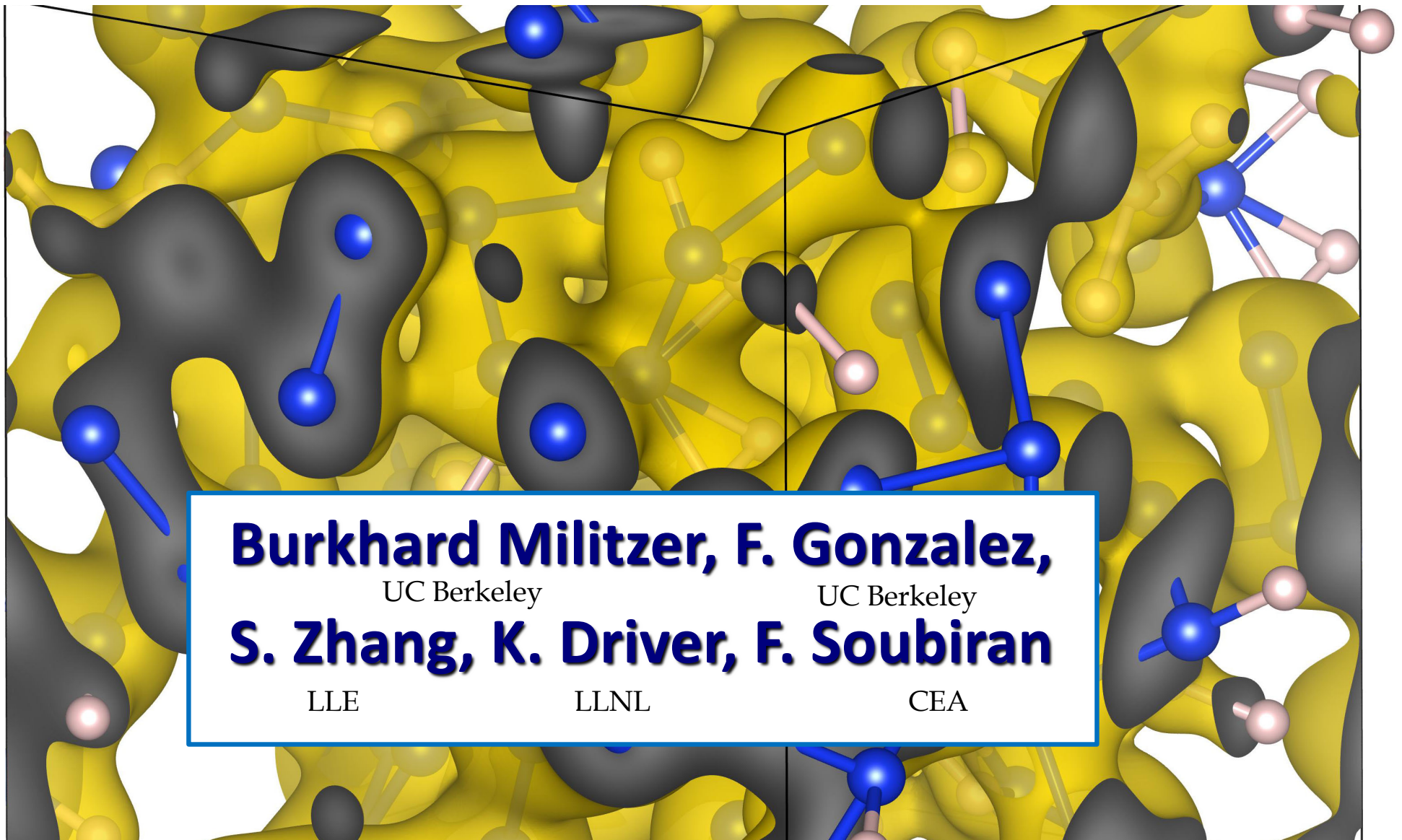
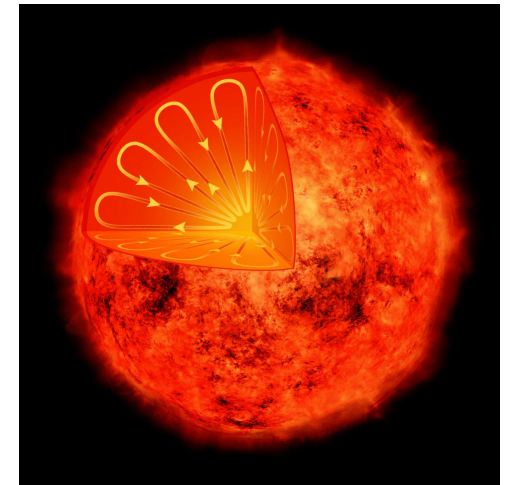
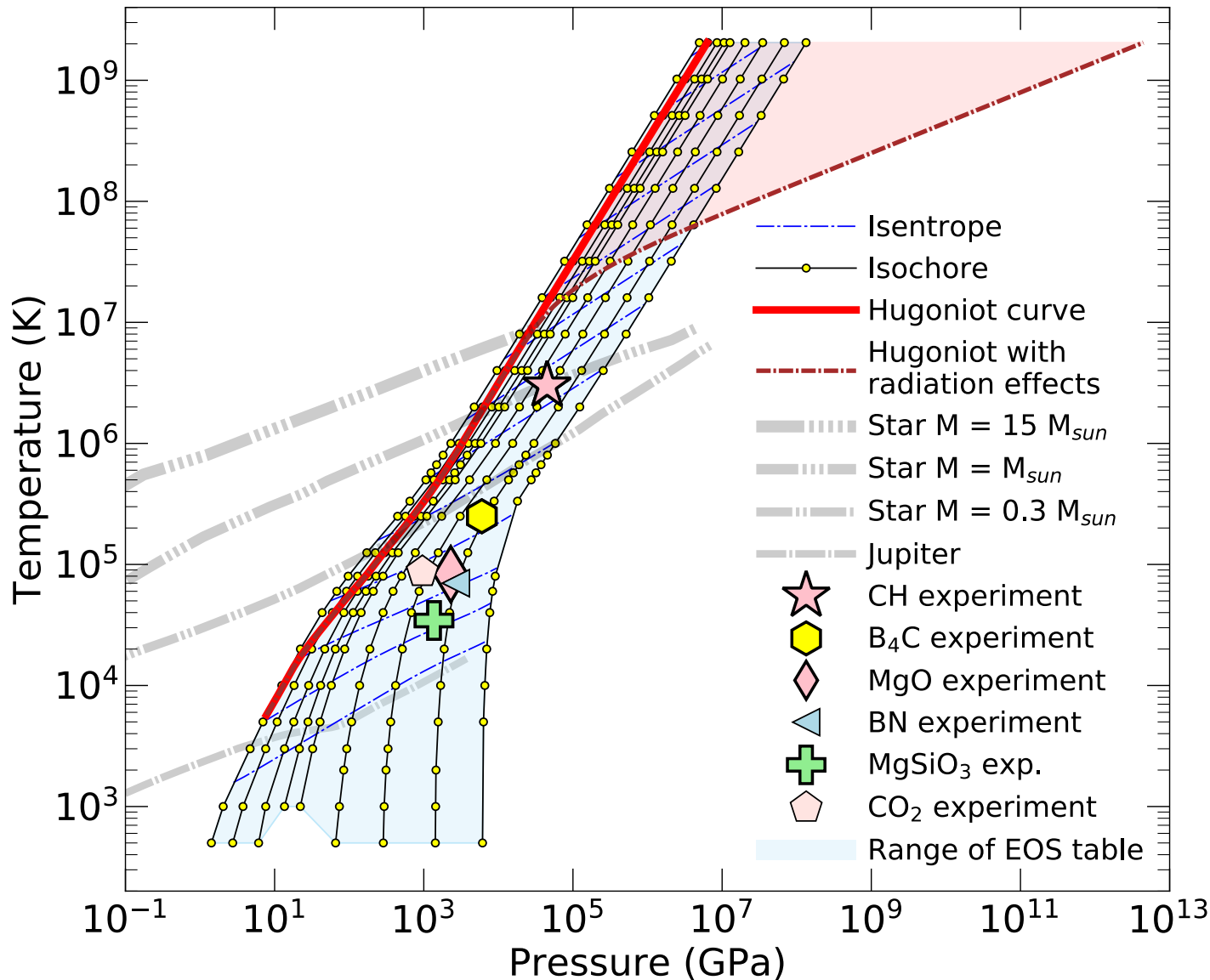


# First-Principles Equation of State Database For Warm Dense Matter Computations





# Outline

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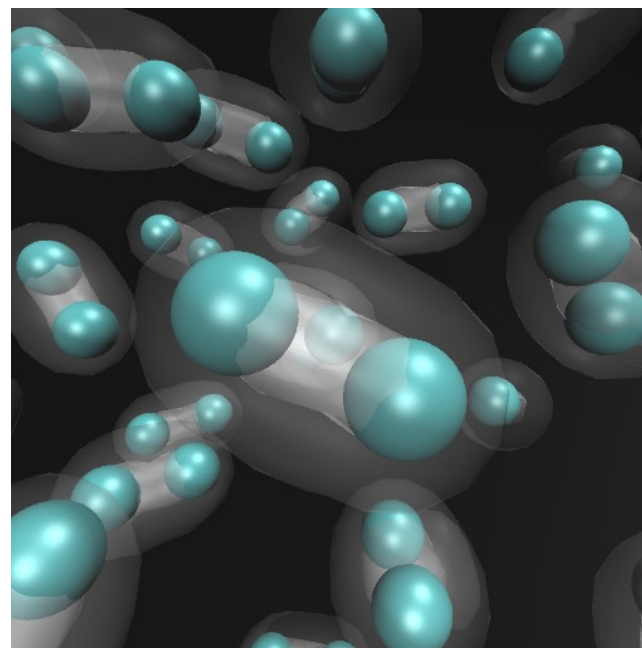
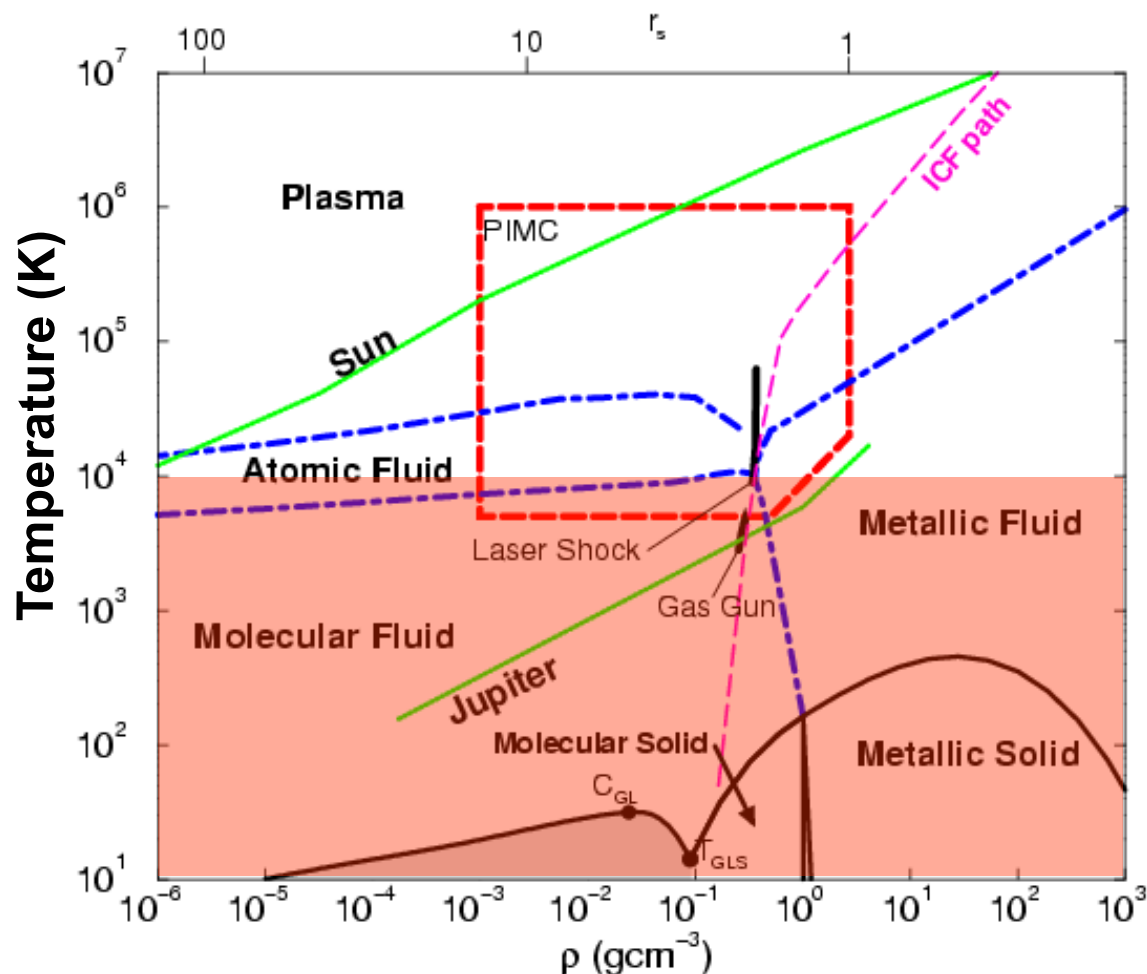
- **Path integral Monte Carlo (PIMC) method**
- **Linear mixing approximation**
- **FPEOS database**
- **Comparison with different experiments**

I.

**Path Integral**

**Monte Carlo**

# Density functional molecular dynamics at lower T

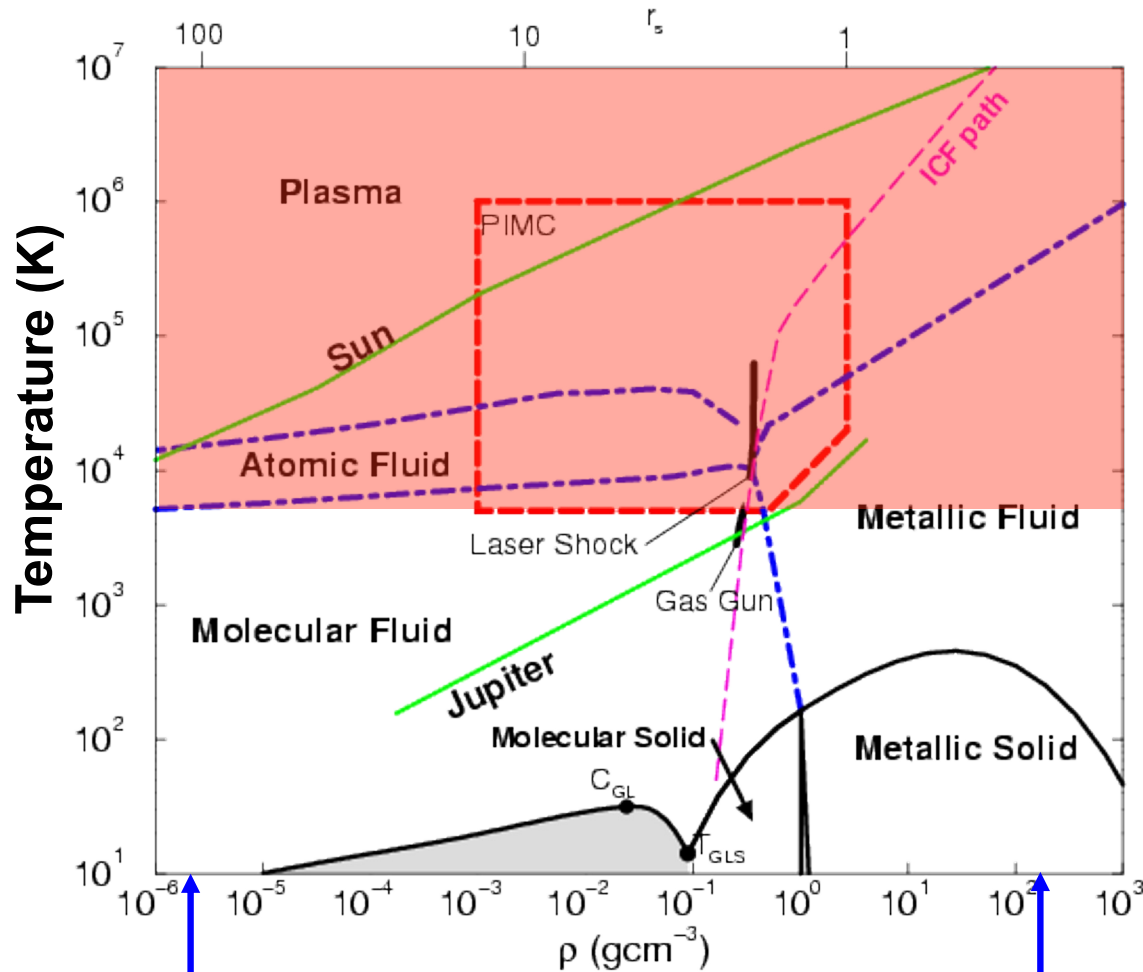


Born-Oppenheimer approx.  
MD with classical nuclei:

$$\mathbf{F} = m \mathbf{a}$$

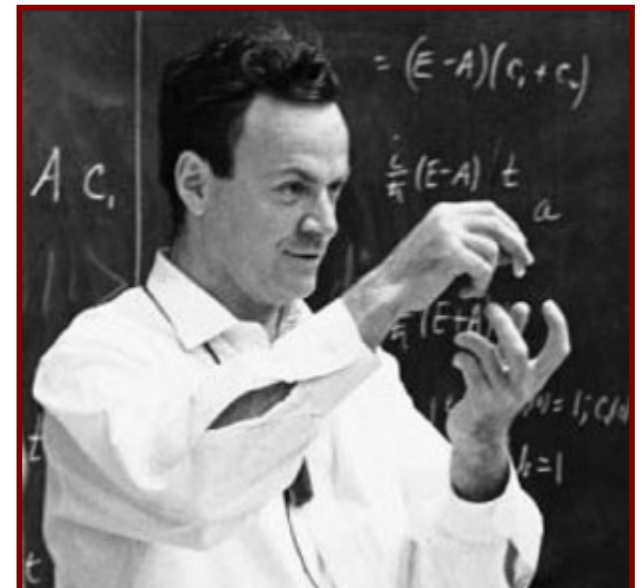
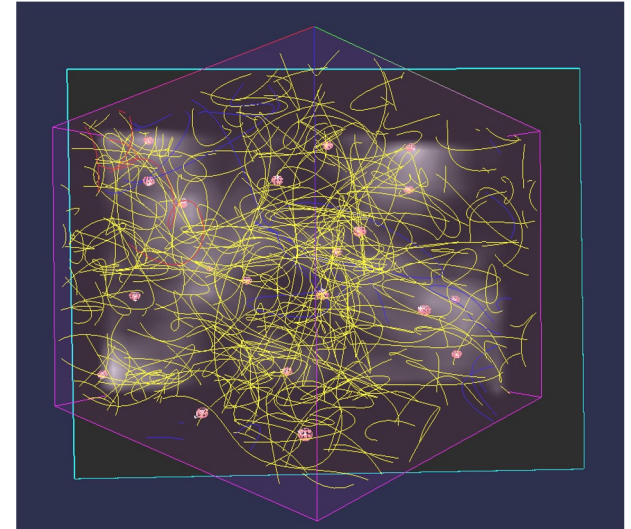
Forces derived DFT with electrons in the instantaneous ground state.

# Path integral Monte Carlo at high $T > 10^4 \dots 10^6$ K



$n=10^{18} \text{ cm}^{-3}$

$n=10^{26} \text{ cm}^{-3}$



# Starting from Restricted PIMC Simulations of Hydrogen

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## PHYSICAL REVIEW LETTERS

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VOLUME 73

17 OCTOBER 1994

NUMBER 16

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### **Equation of State of the Hydrogen Plasma by Path Integral Monte Carlo Simulation**

C. Pierleoni,<sup>1,2,\*</sup> D. M. Ceperley,<sup>3</sup> B. Bernu,<sup>1</sup> and W. R. Magro<sup>3</sup>

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VOLUME 76, NUMBER 8

PHYSICAL REVIEW LETTERS

19 FEBRUARY 1996

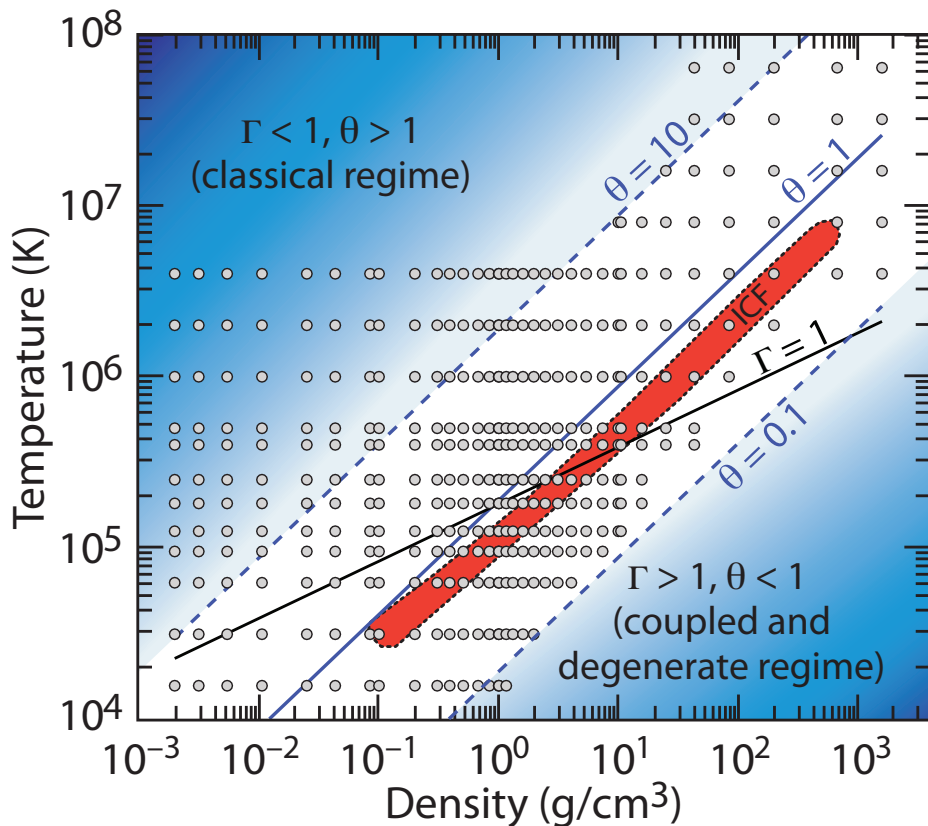
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### **Molecular Dissociation in Hot, Dense Hydrogen**

W. R. Magro,<sup>1</sup> D. M. Ceperley,<sup>2</sup> C. Pierleoni,<sup>3</sup> and B. Bernu<sup>4</sup>

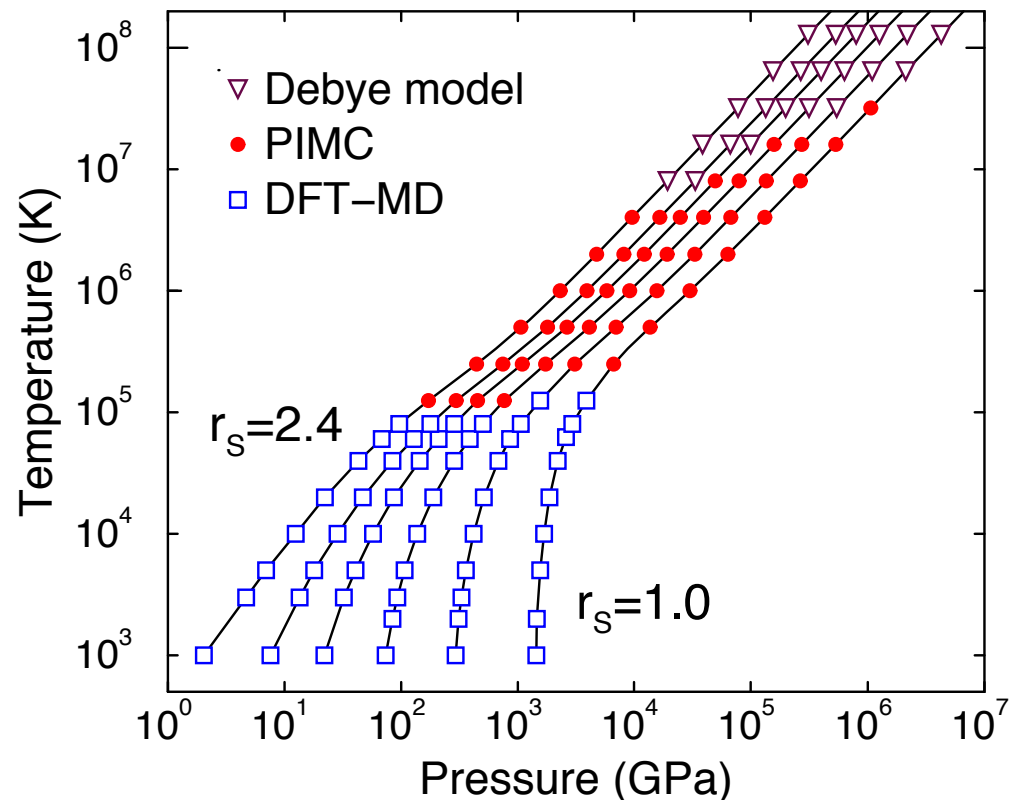
# PIMC and DFT-MD Simulations of Hydrogen and Helium

## Hydrogen



S. X. Hu, B. Militzer, V. N. Goncharov, S. Skupsky, *Phys. Rev. Lett.*, **104** (2010) 235003  
*Phys. Rev. B* **84** (2011) 224109.

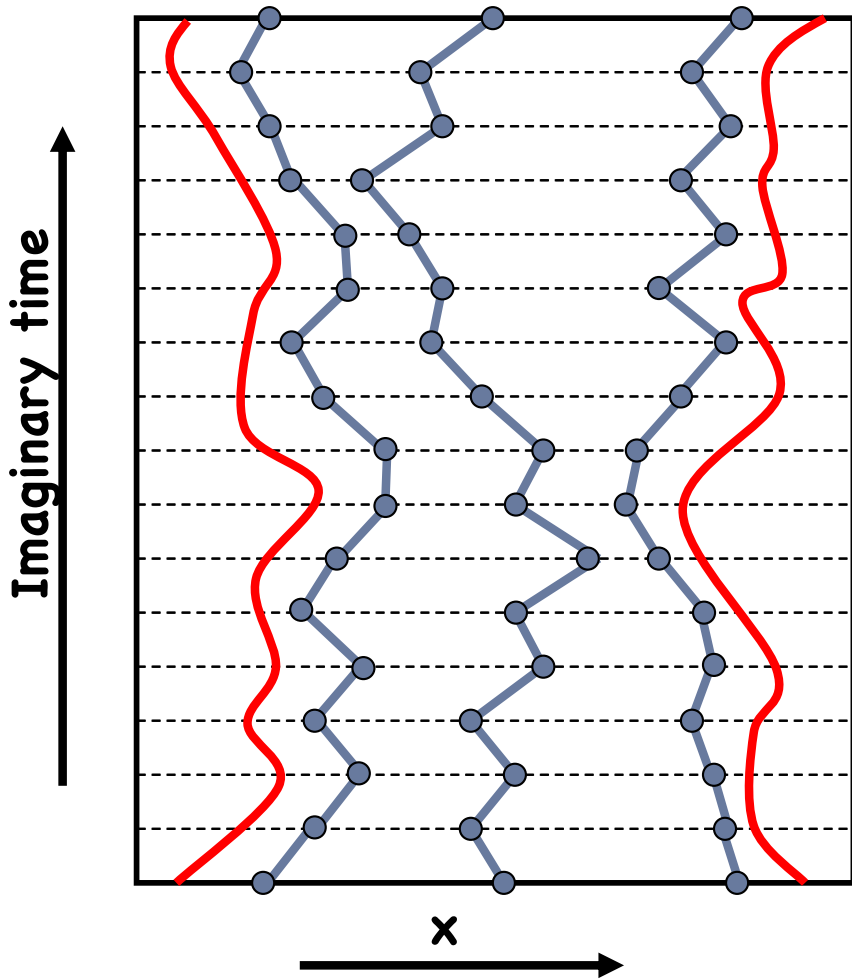
## Helium



B. Militzer, *Phys. Rev. B* **79** (2009) 155105  
 B. Militzer, *Phys. Rev. Lett.* **97** (2006) 175501



# Restricted PIMC for fermions: How is the restriction applied?



Construct a **fermionic trial density matrix** in form of a Slater determinant of single-particle density matrices:

$$\rho_T(R, R', \beta) = \begin{vmatrix} \rho(r_1, r'_1, \beta) & \cdots & \rho(r_1, r'_N, \beta) \\ \vdots & \ddots & \vdots \\ \rho(r_N, r'_1, \beta) & \cdots & \rho(r_N, r'_N, \beta) \end{vmatrix}$$

Enforce the following nodal condition for all time slices along the paths:

$$\rho_T[R(t), R(0), t] > 0$$

This 3N-dimensional conditions eliminates all negative and some positive contribution to the path → Solves the fermion sign problem approx.

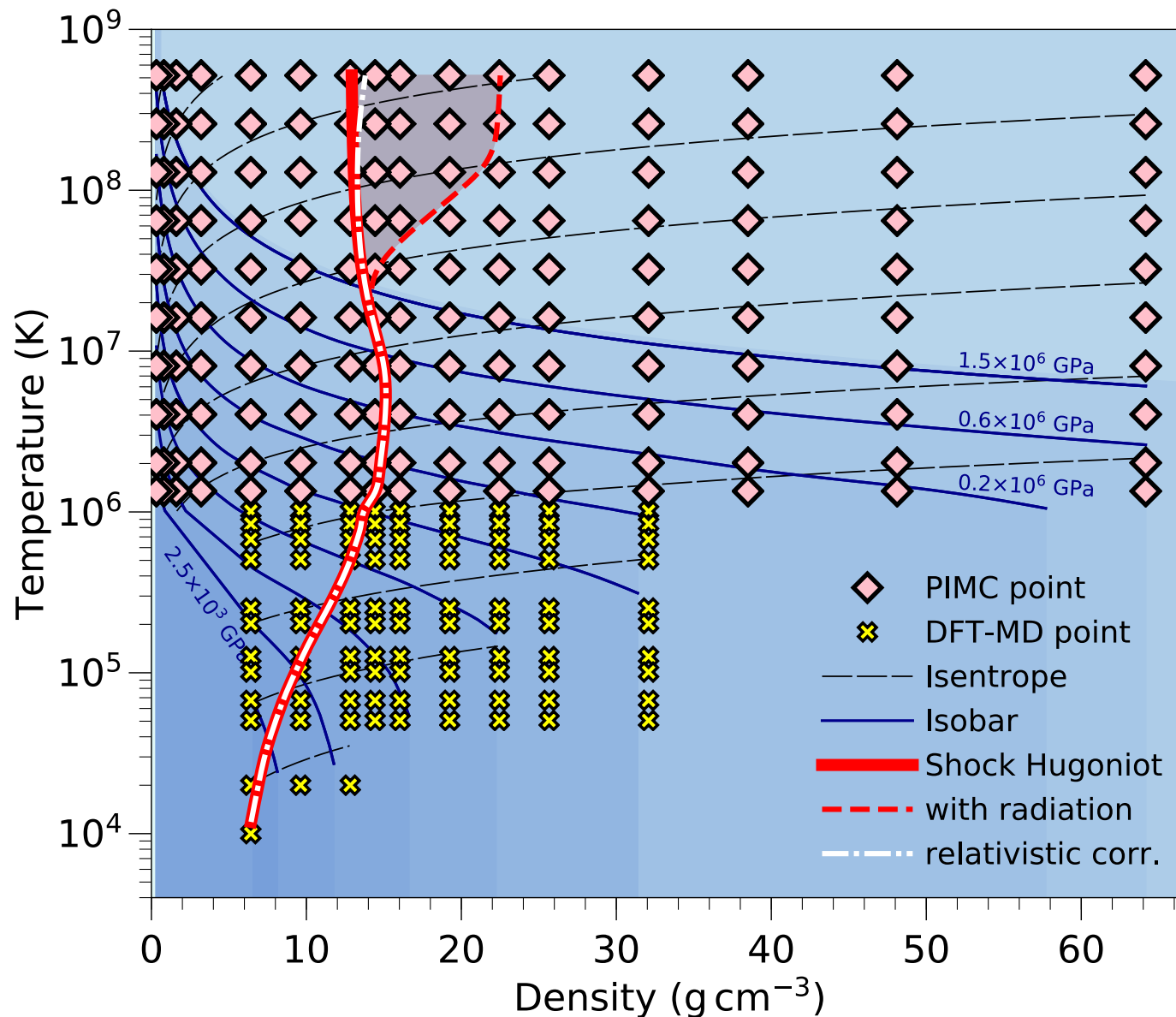
Free-particle nodes:

$$\rho_0^{[1]}(r, r'; \beta) = \sum_k e^{-\beta E_k} \Psi_k(r) \Psi_k^*(r')$$

**Silicates:**

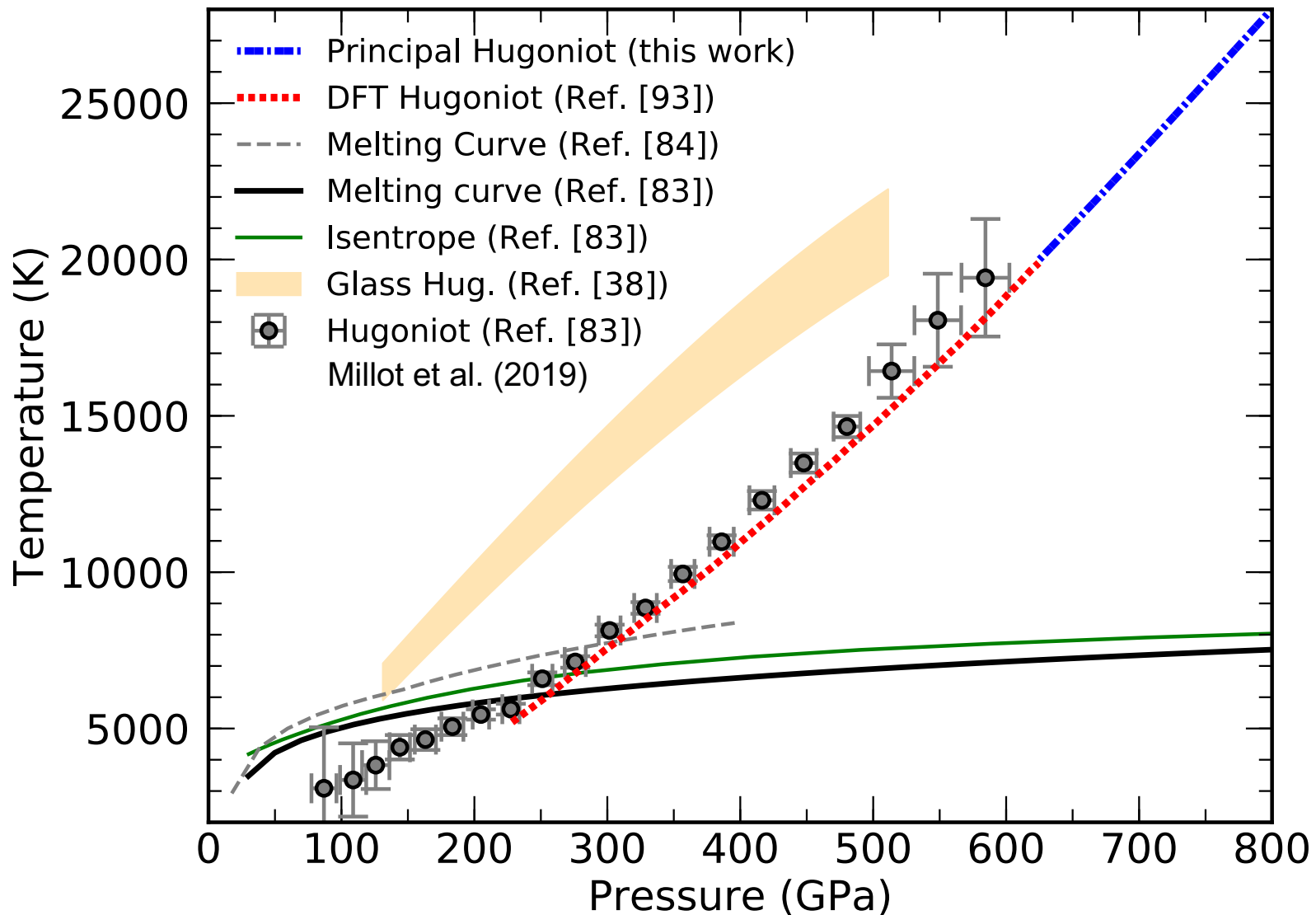


# MgSiO<sub>3</sub> : Principal Hugoniot Curve

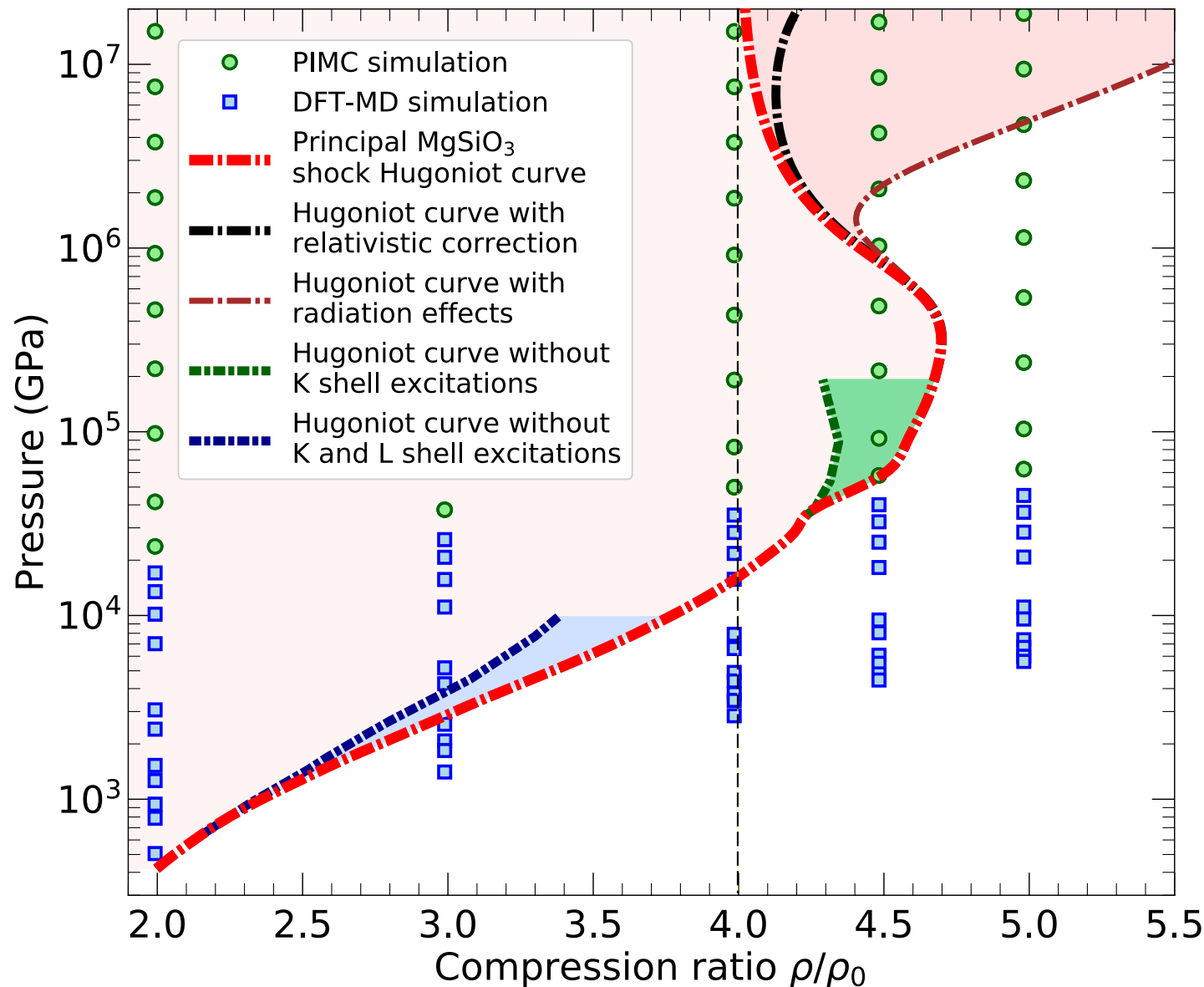


Gonzalez,  
Soubiran,  
Peterson,  
Militzer,  
[Phys. Rev. B](#)  
**101** (2020)  
024107

# MgSiO<sub>3</sub> : Principal Hugoniot Curve



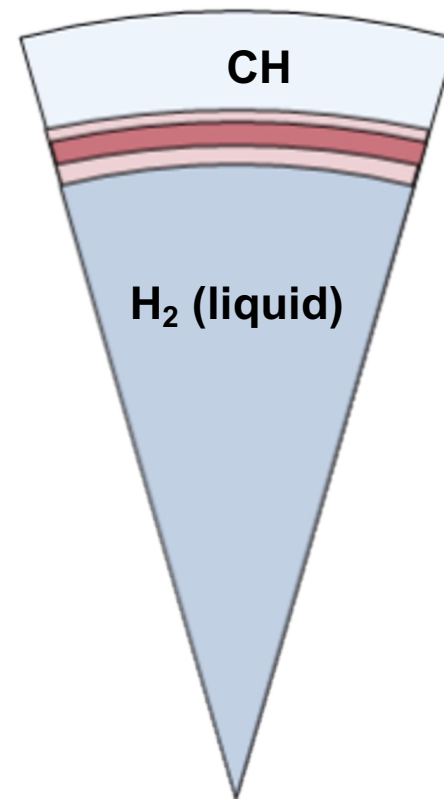
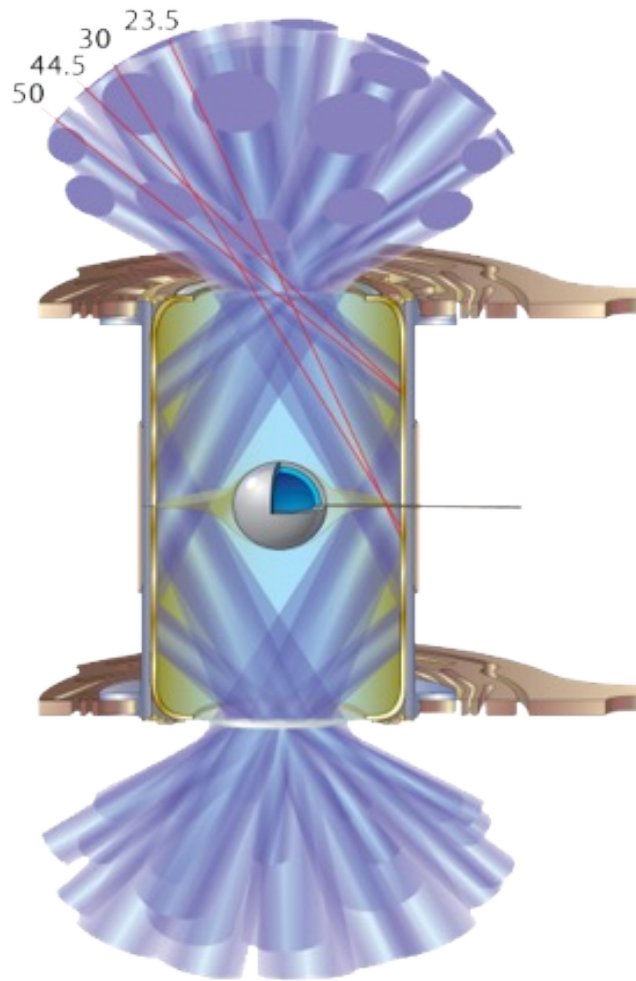
# MgSiO<sub>3</sub> : Principal Hugoniot Curve



Gonzalez,  
Soubiran,  
Peterson,  
Militzer,  
[Phys. Rev. B](#)  
**101** (2020)  
024107

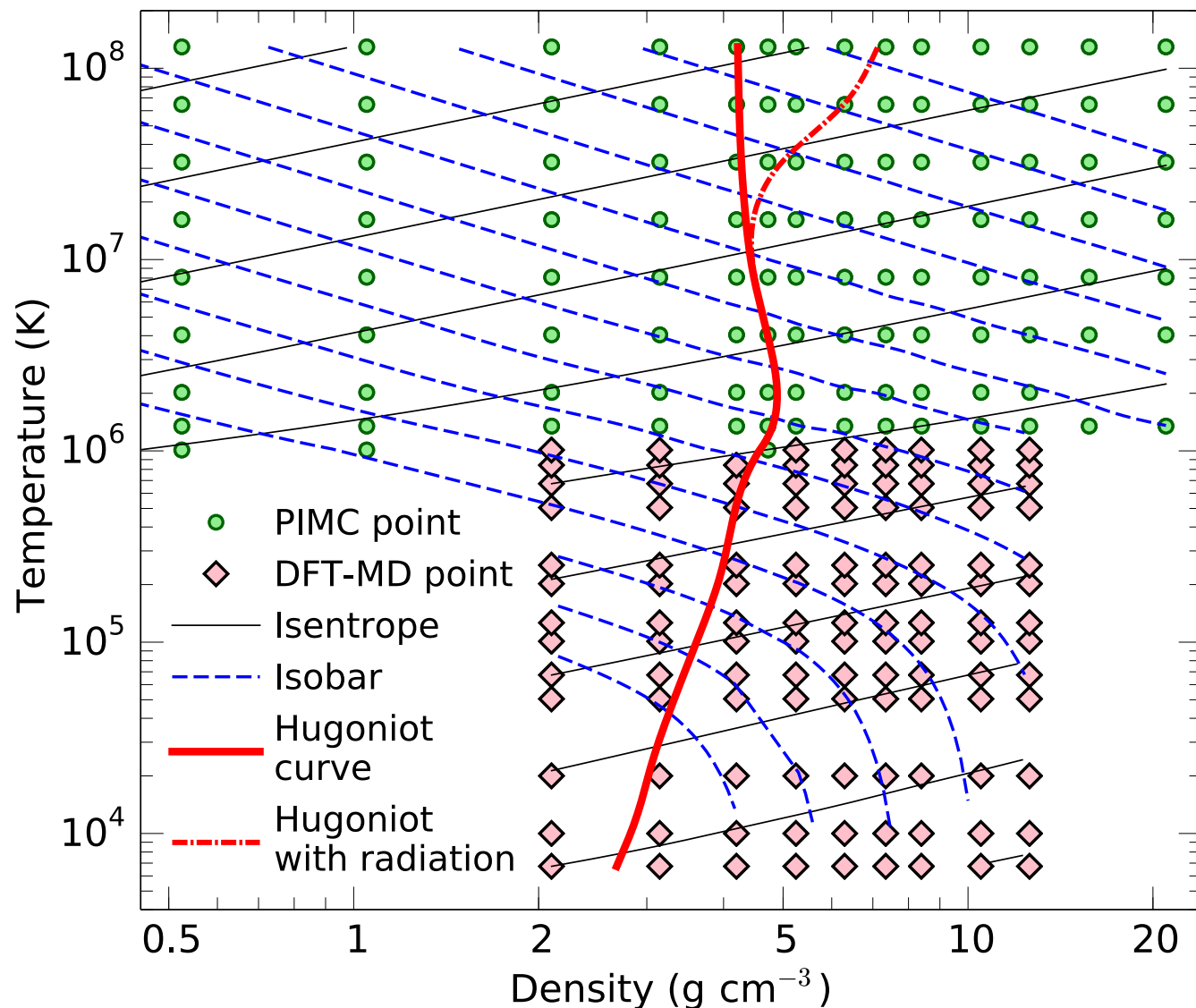
***CH plastics***

# Inertial confinement fusion experiments with plastic coated spheres of liquid H<sub>2</sub>



(Graphics: Bachmann et al. LLNL)

# PIMC and DFT-MD simulations performed for $C_2H$ , $CH$ , $C_2H_3$ , $CH_3$ and $CH_4$ .



All calculations  
performed on

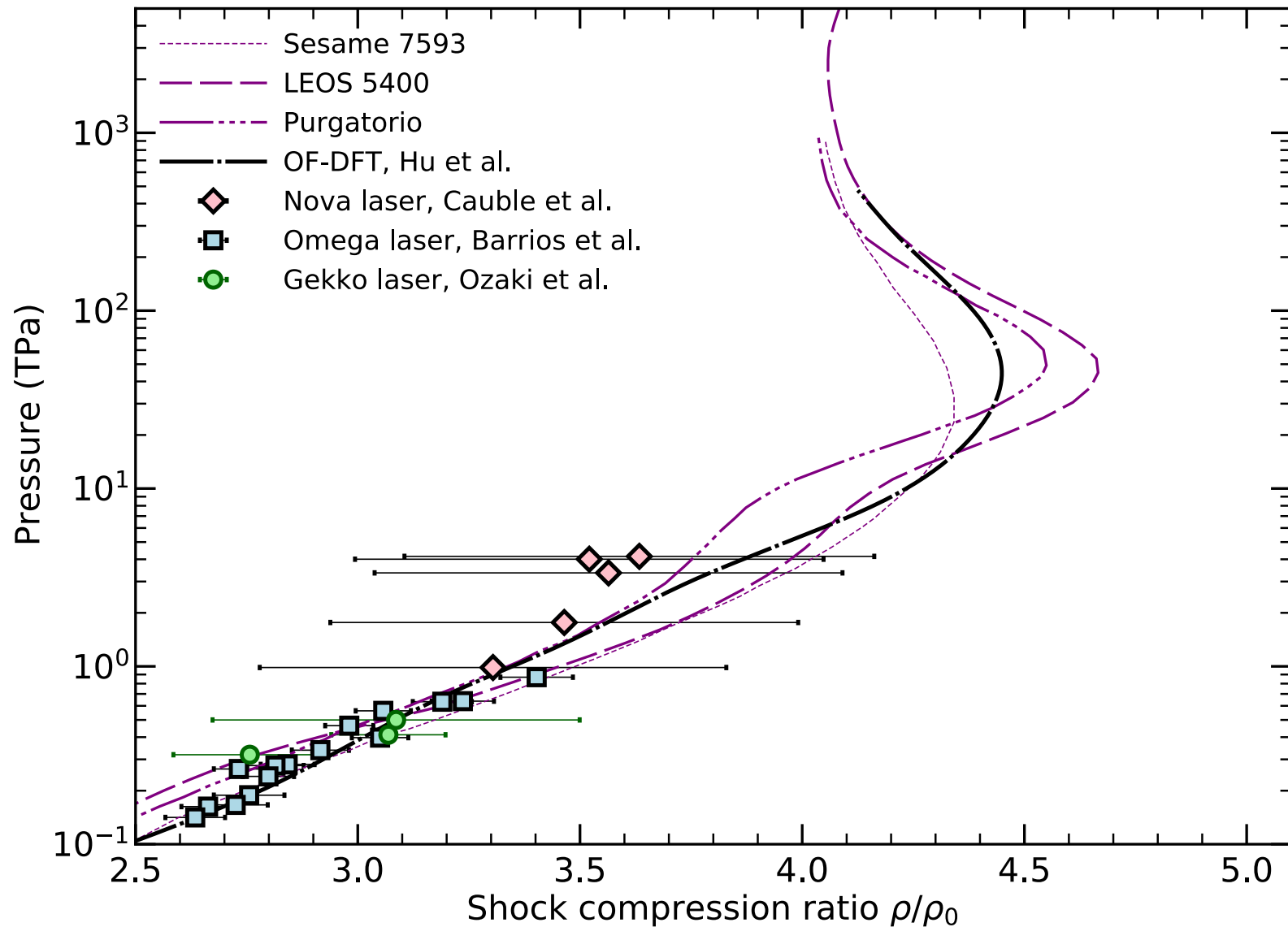
**BLUE WATERS**  
SUSTAINED PETASCALE COMPUTING

**NCSA**

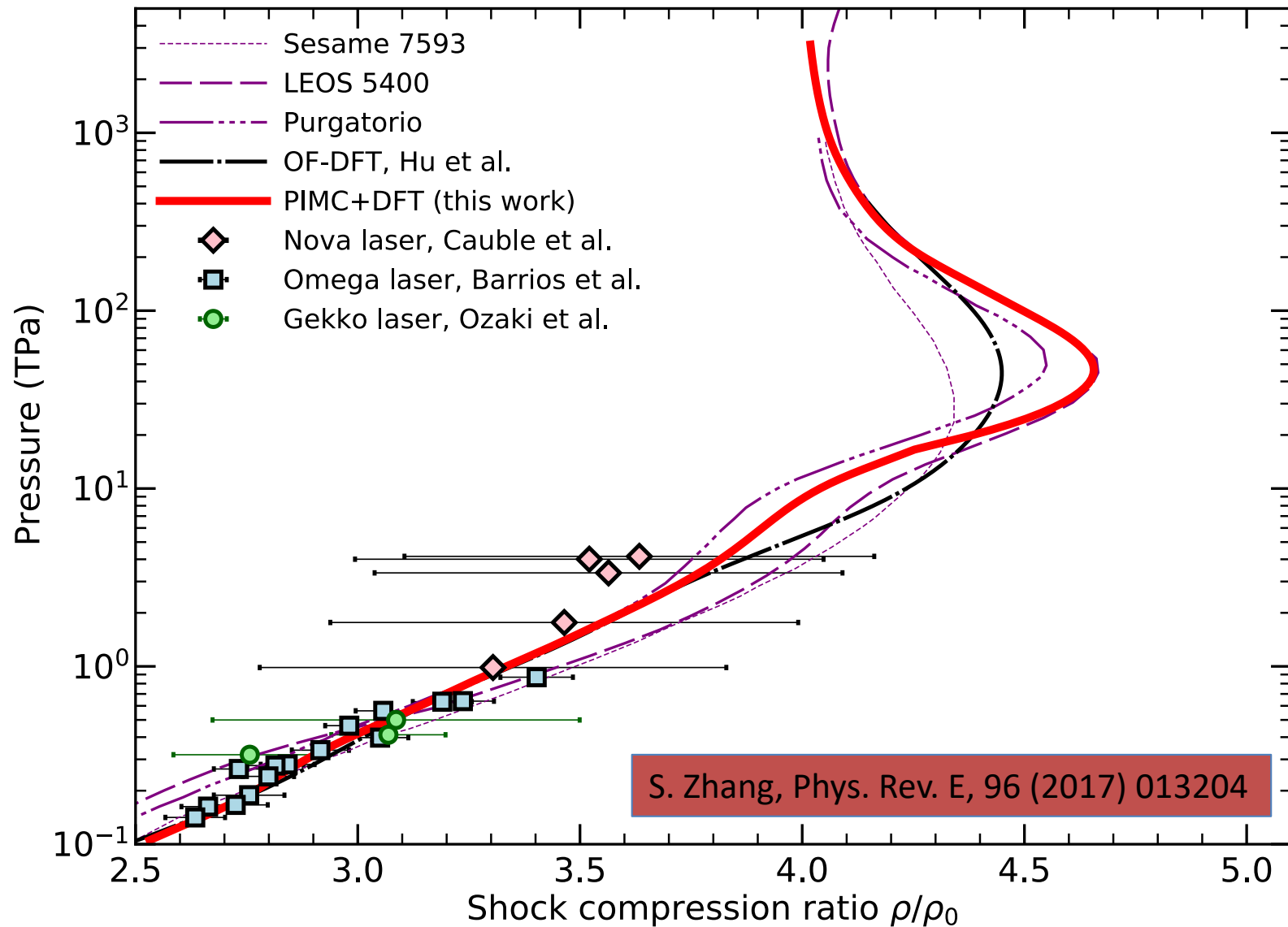
**I**



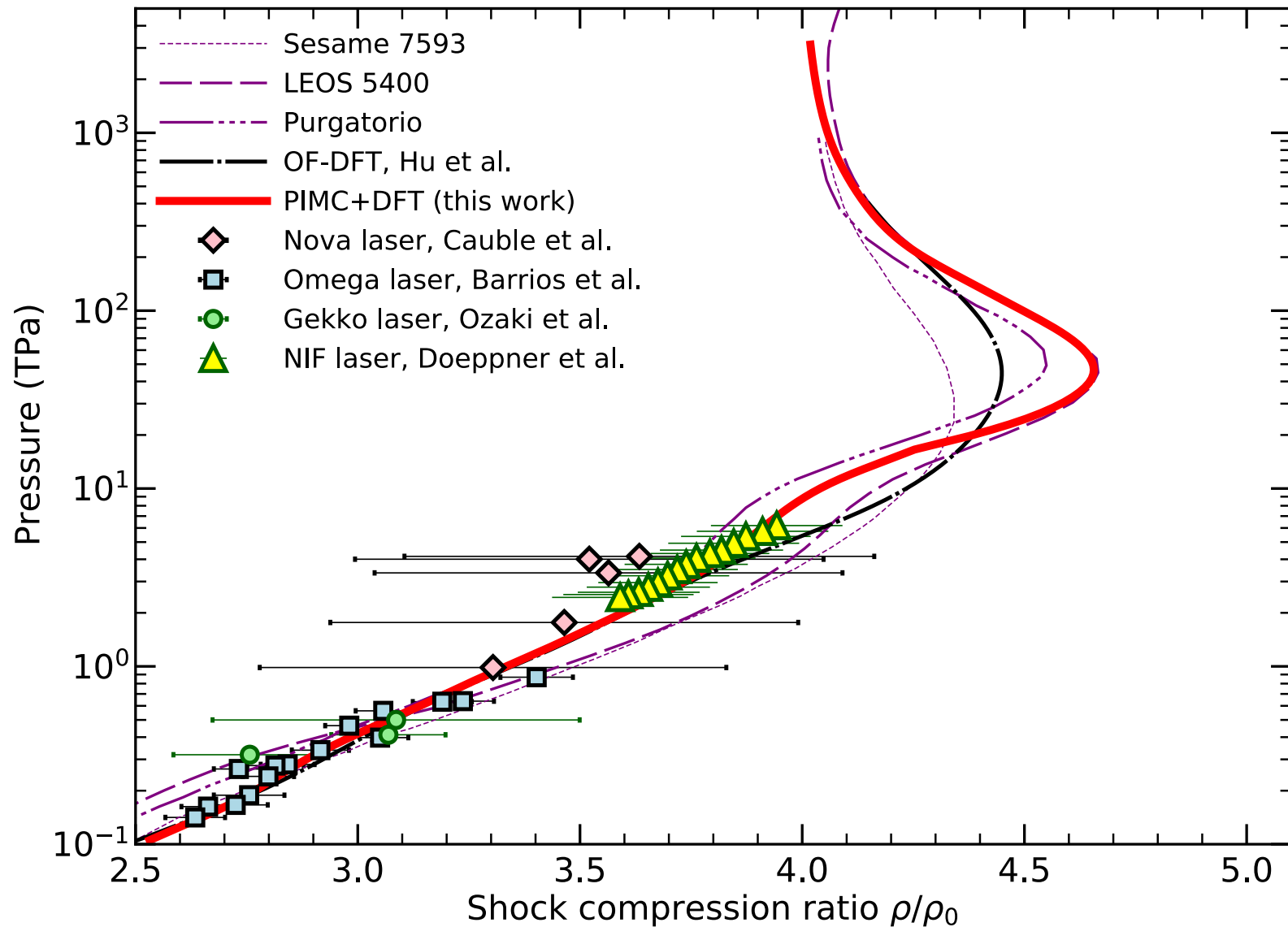
# CH Shock Hugoniot Curves: Comparison of Theory and Experiments



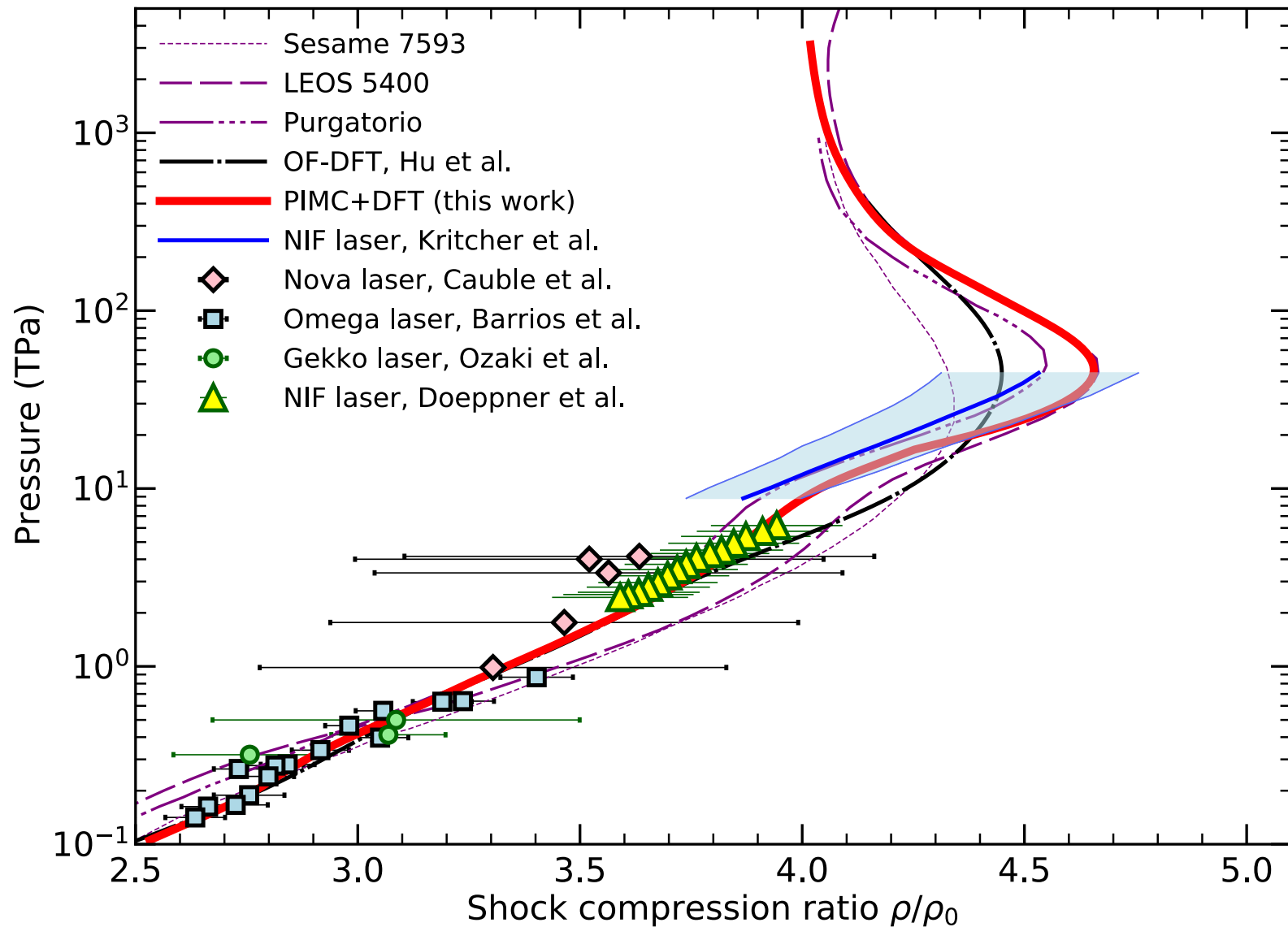
# CH Shock Hugoniot Curves: Comparison of Theory and Experiments



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# CH Shock Hugoniot Curves: Comparison of Theory and Experiments

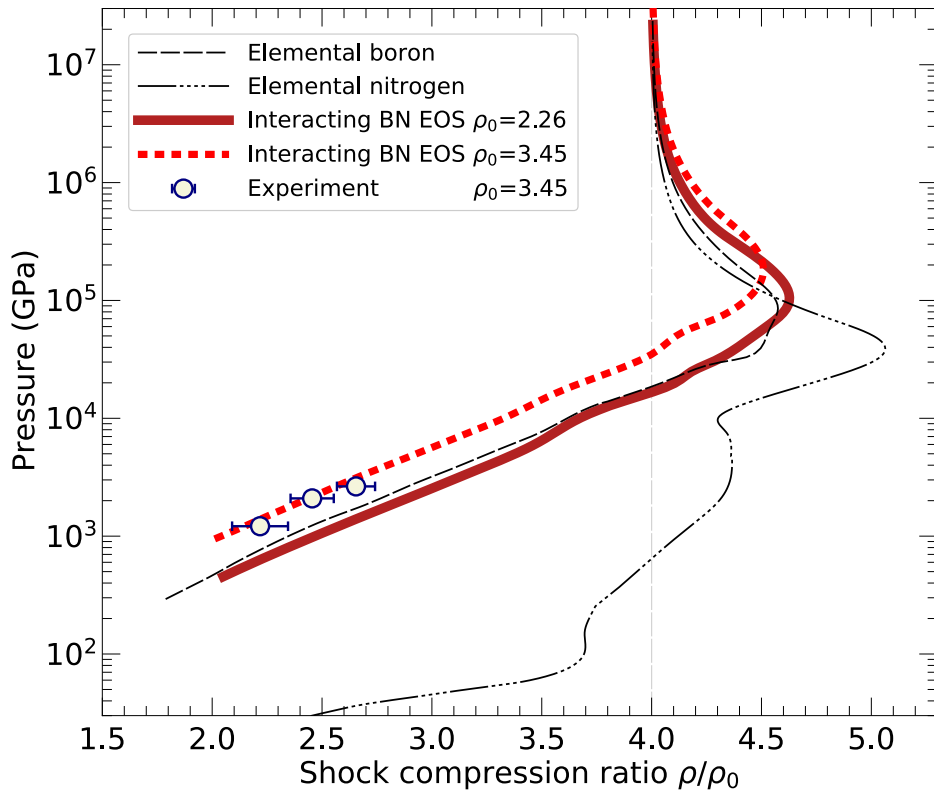


***Linear Mixing  
and  
FPEOS database***

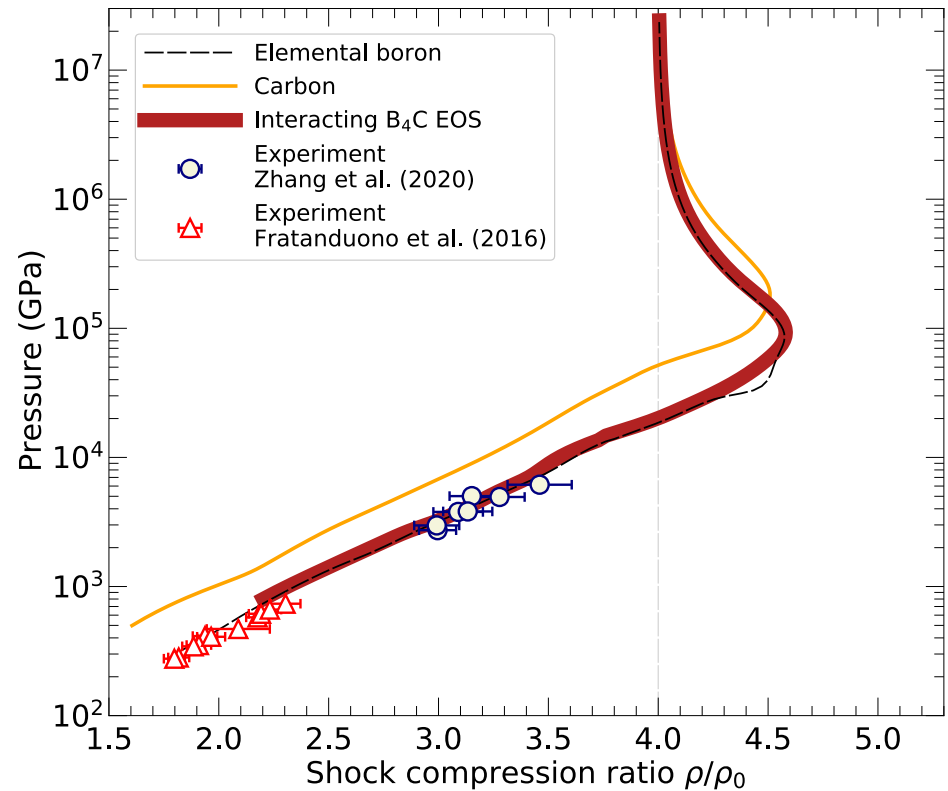
# Hugoniot Curves of **BN** and **B<sub>4</sub>C**

Fully interacting EOS and Linear Mixing agree quite well.

Boron nitride  
Zhang et al. PRB 2019



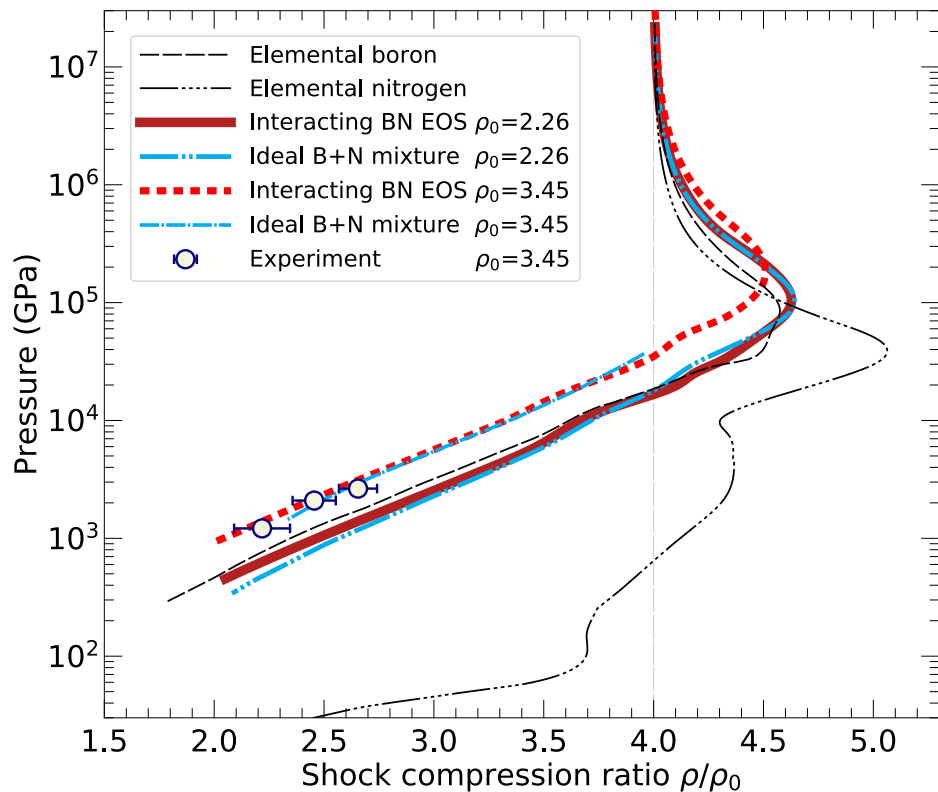
Boron carbide  
Zhang et al. PRE 2020



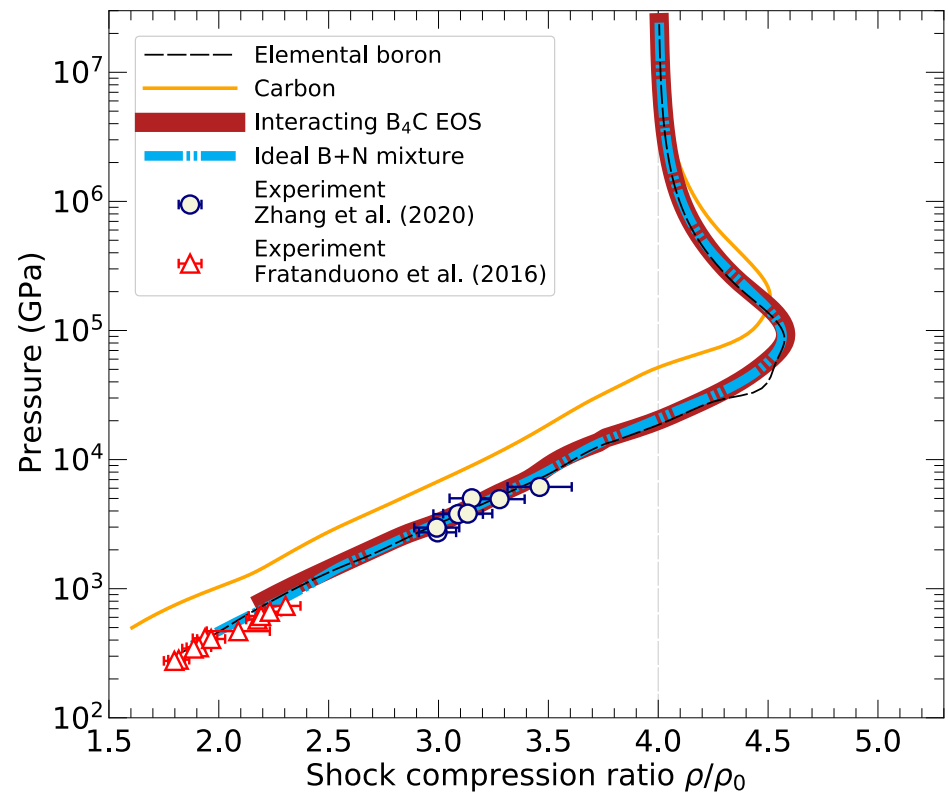
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Boron nitride  
Zhang et al. PRB 2019

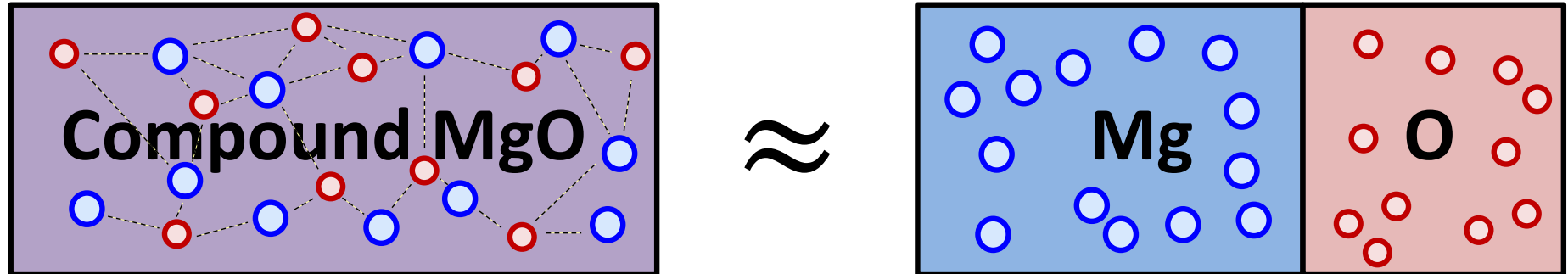


Boron carbide  
Zhang et al. PRE 2020



# Linear Mixing at Constant P and T

(Also called additive volume rule)



$$V_{\text{mix}} = N_1 V_1 + N_2 V_2 ,$$

$$m_{\text{mix}} = N_1 m_1 + N_2 m_2 ,$$

$$E_{\text{mix}} = N_1 E_1 + N_2 E_2 ,$$

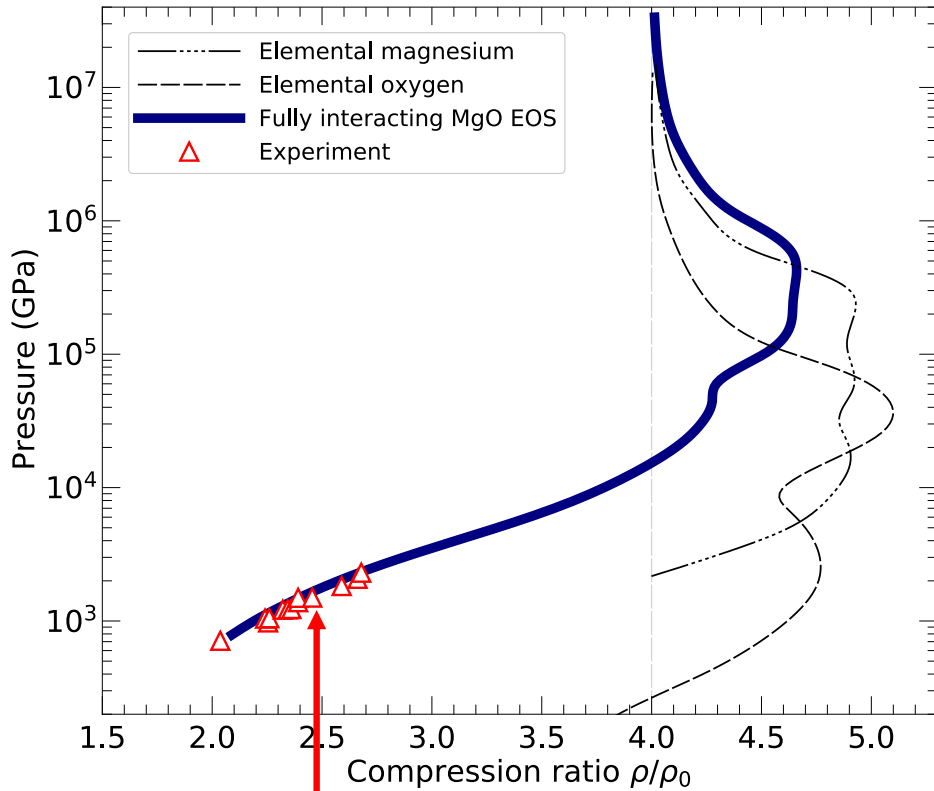
$$\rho_{\text{mix}} = m_{\text{mix}} / V_{\text{mix}}$$



# Hugoniot Curves of $\text{MgO}$ and $\text{MgSiO}_3$

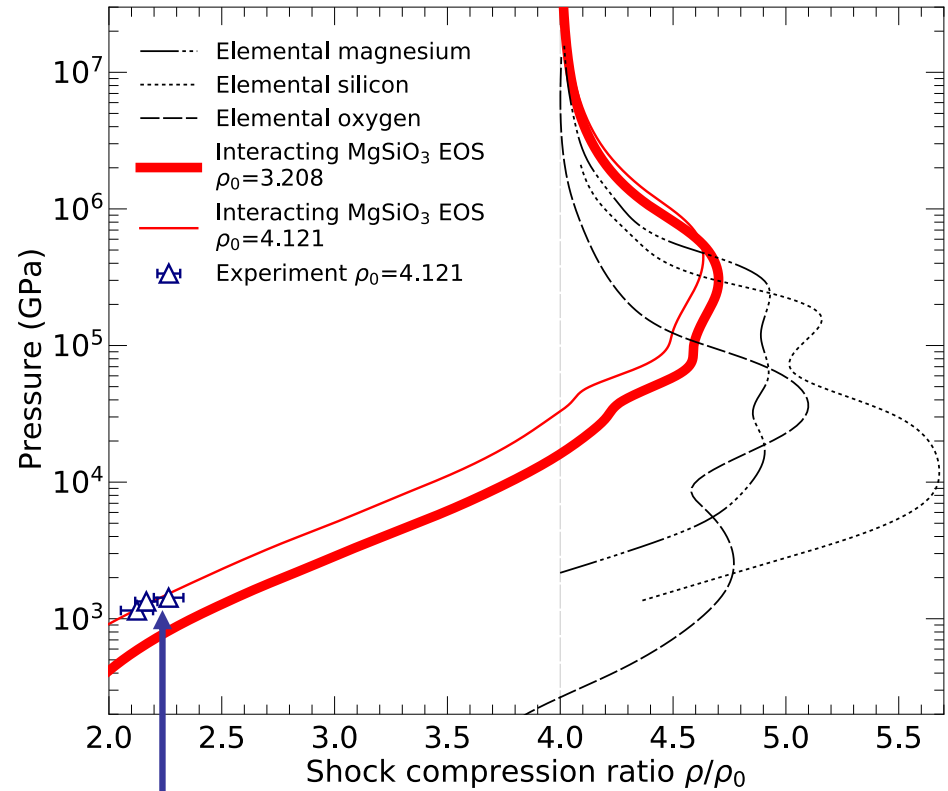
Results from fully interacting EOS and experiment.

Soubiran et al. JCP 2019



McCoy et al.  
PRB 2019

Gonzalez et al. PRB 2020

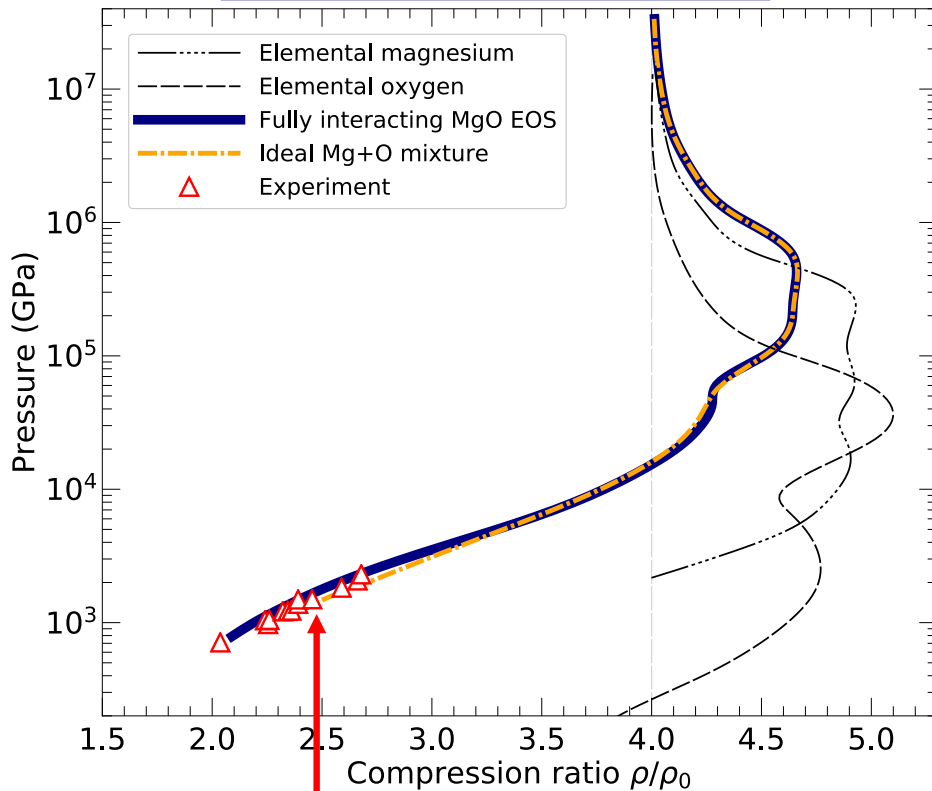


Millot et al.  
GRL 2020

# Hugoniot Curves of **MgO** and **MgSiO<sub>3</sub>**

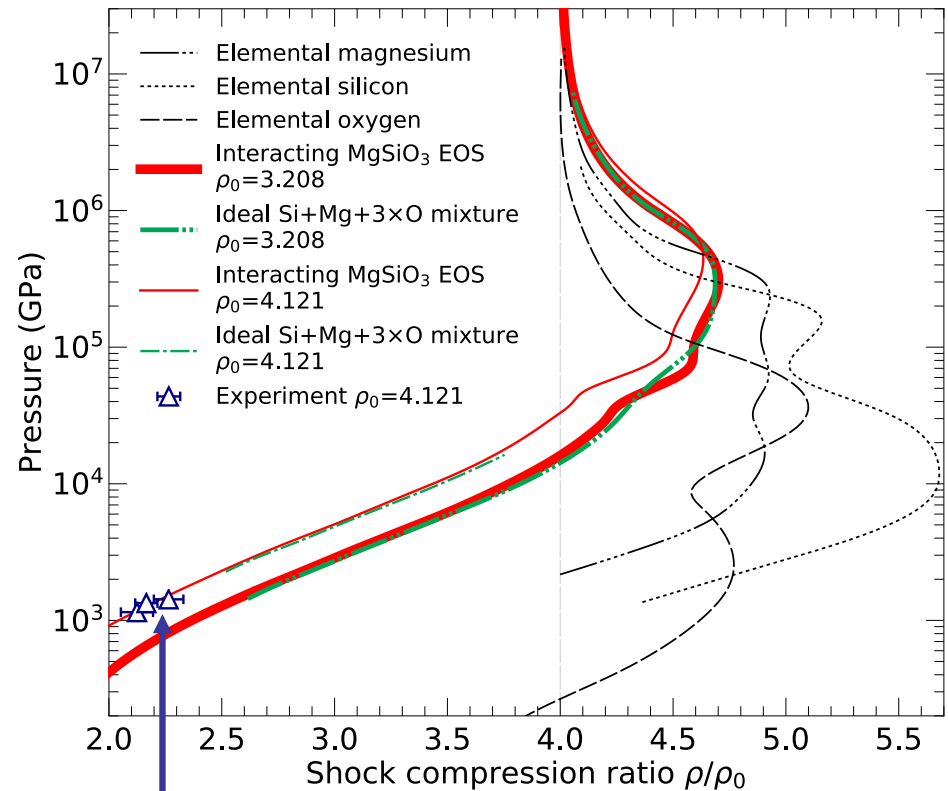
Fully interacting EOS and Linear Mixing agree quite well.

Soubiran et al. JCP 2019



McCoy et al.  
PRB 2019

Gonzalez et al. PRB 2020



Millot et al.  
GRL 2020

Linear mixing works well for  $T \gtrsim 2 \times 10^5 \text{ K}$  and  $\rho/\rho_0 \gtrsim 3.2$

# Nonideal mixing effects in warm dense matter studied with first-principles computer simulations

Cite as: J. Chem. Phys. 153, 184101 (2020); doi: [10.1063/5.0023232](https://doi.org/10.1063/5.0023232)

Submitted: 28 July 2020 • Accepted: 25 October 2020 •

Published Online: 9 November 2020



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Export Citation

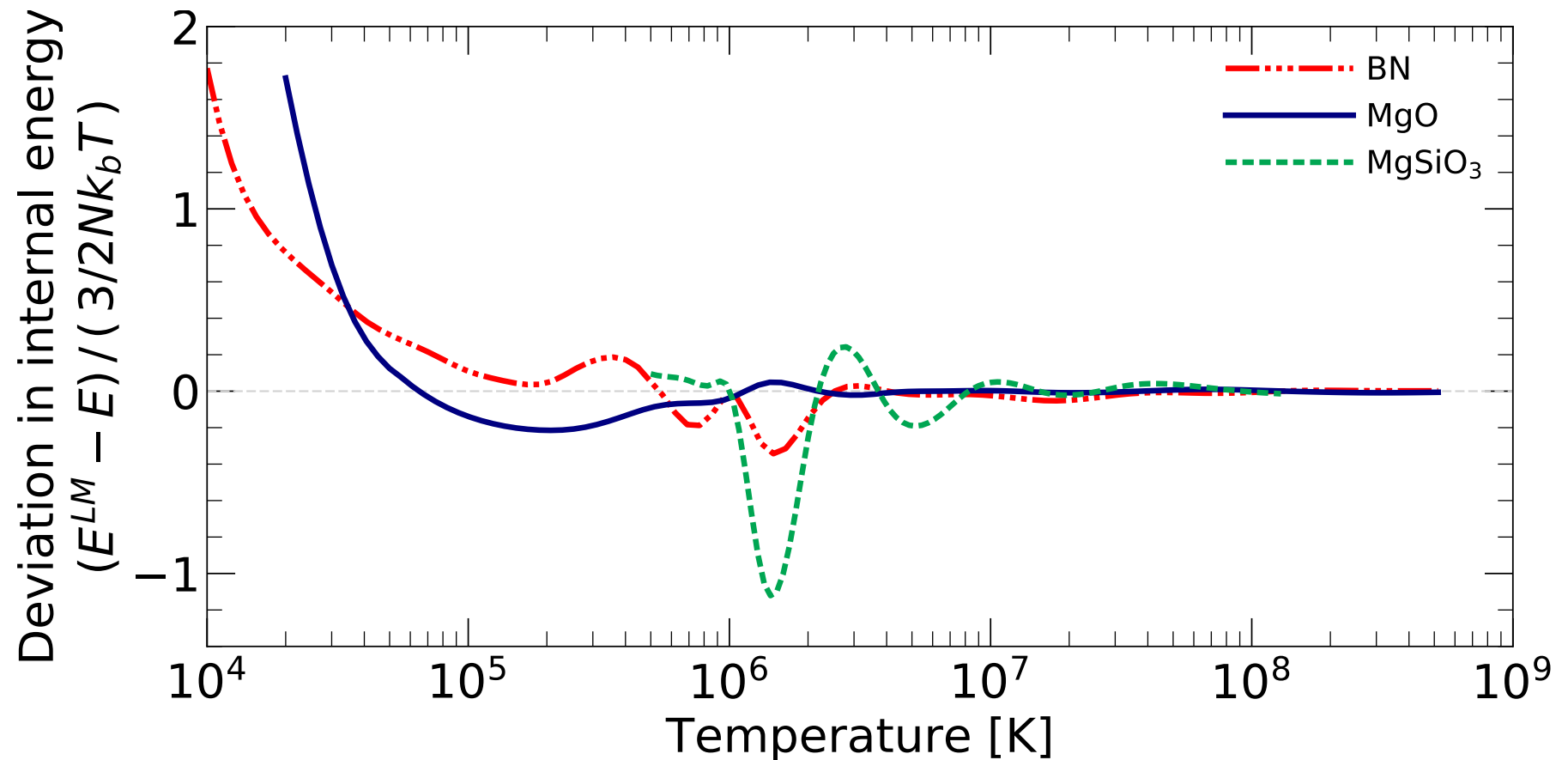


CrossMark

Burkhard Militzer,<sup>1,2,a)</sup>  Felipe González-Cataldo,<sup>1</sup>  Shuai Zhang,<sup>3</sup>  Heather D. Whitley,<sup>4</sup>  Damian C. Swift,<sup>4</sup>  
and Marius Millot<sup>4</sup> 

# Nonlinear Mixing Effects in $\text{MgSiO}_3$

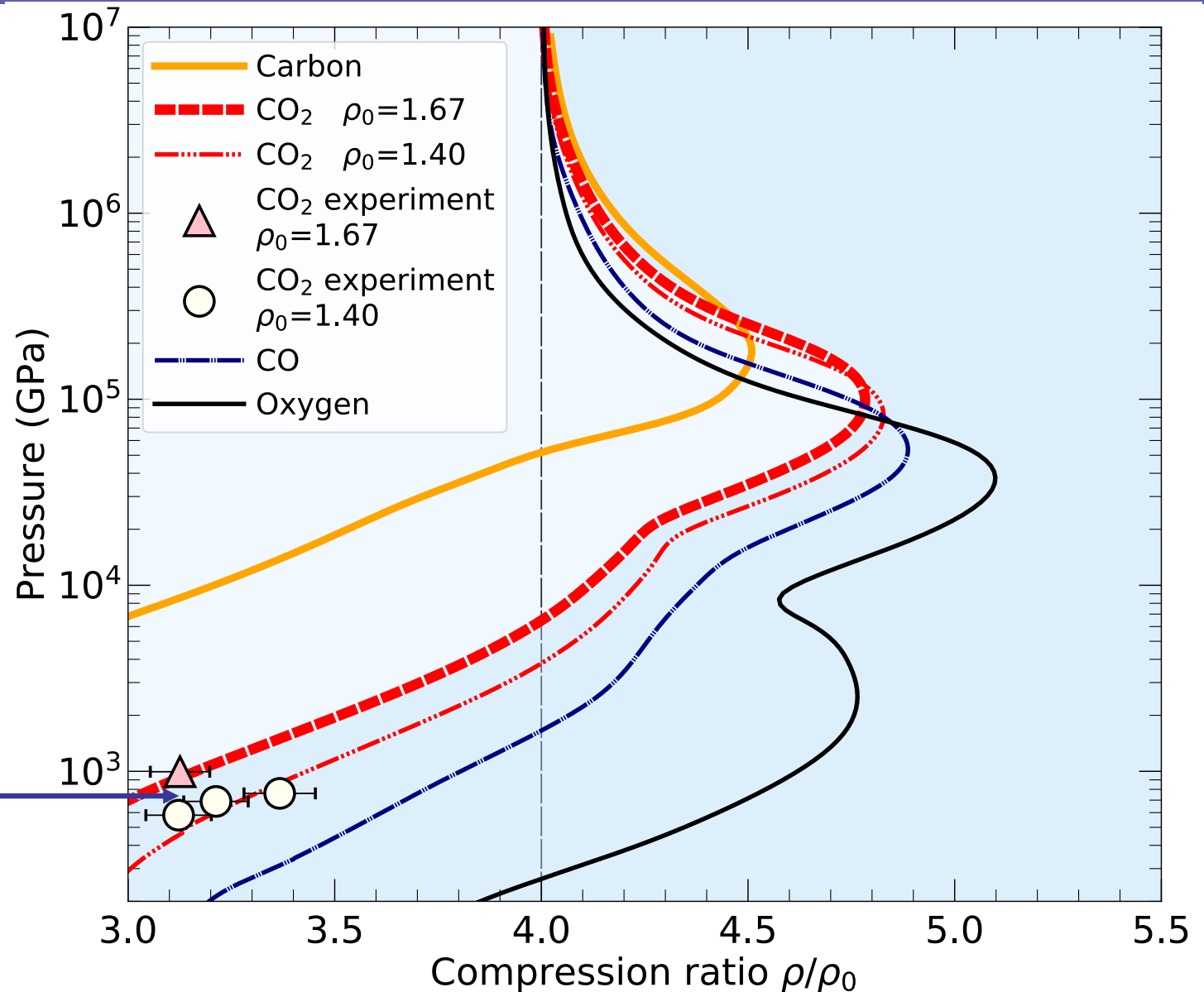
Fully interacting EOS and Linear Mixing agree quite well.



Linear mixing works well for  $T \gtrsim 2 \times 10^5 \text{ K}$  and  $\rho/\rho_0 \gtrsim 3.2$

# Hugoniot Curves of **CO** and **CO<sub>2</sub>**

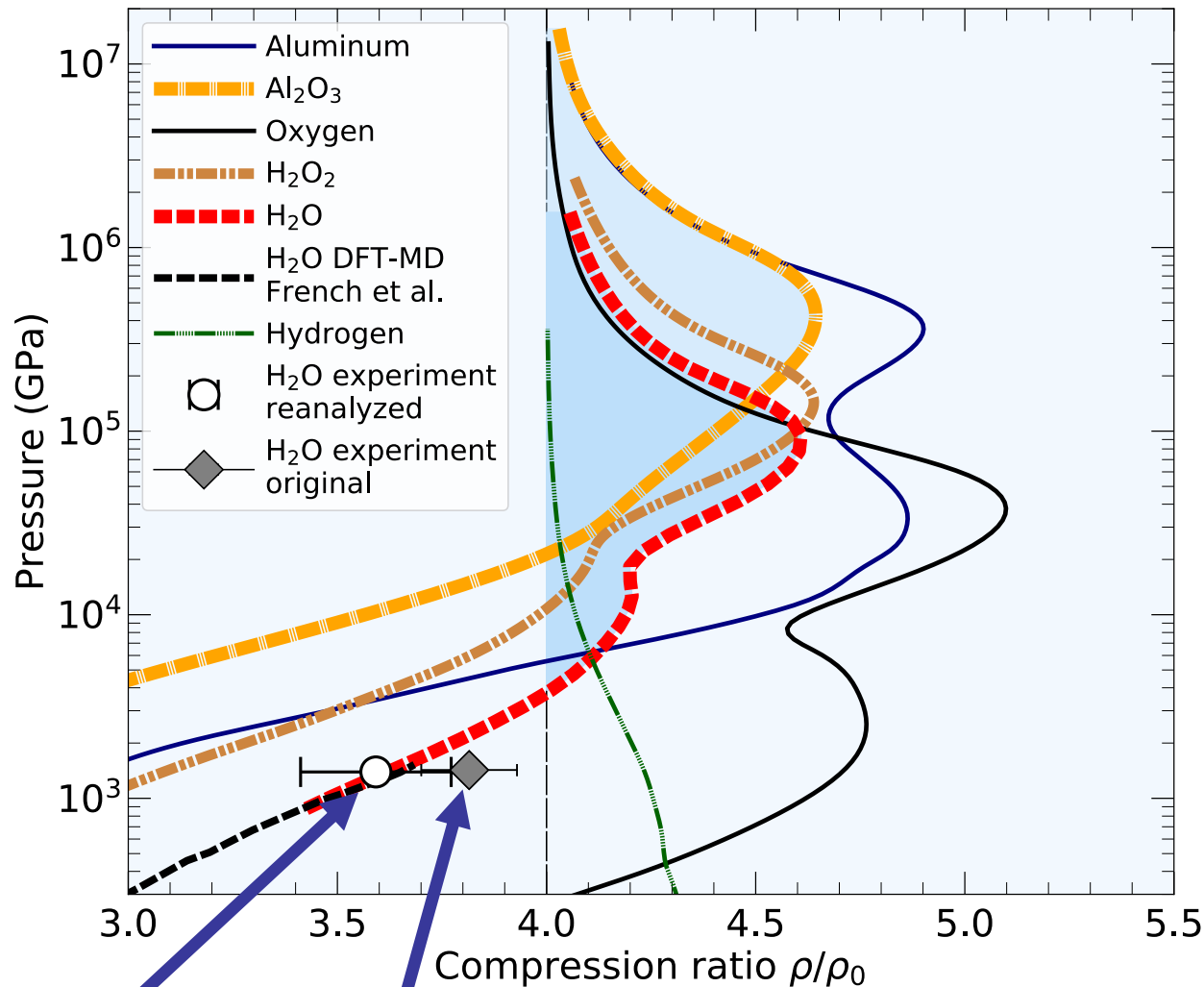
Experimental **CO<sub>2</sub>** Hugoniot agree with Linear Mixing result



Crandall et al.  
PRL 2020

# Hugoniot Curves of $\text{H}_2\text{O}$ , $\text{H}_2\text{O}_2$ , and $\text{Al}_2\text{O}_3$

## Experimental $\text{H}_2\text{O}$ Hugoniot agree with Linear Mixing result



Reanalyzed by  
Knudson et al.

Experiments by  
Podurets et al. 1972



# FPEOS: 11+10 Available Tables



Material	Number of isochores	Minimum density [g cm <sup>-3</sup> ]	Maximum density [g cm <sup>-3</sup> ]	Minimum temperature [K]	Maximum temperature [K]	Number of EOS points	References
Hydrogen	33	0.001	798.913	15625	6.400×10 <sup>7</sup>	401	[69–74]
Helium	9	0.387	10.457	500	2.048×10 <sup>9</sup>	228	[75, 76]
Boron	16	0.247	49.303	2000	5.174×10 <sup>8</sup>	314	[77]
Carbon	9	0.100	25.832	5000	1.035×10 <sup>9</sup>	162	[78, 79]
Nitrogen	17	1.500	13.946	1000	1.035×10 <sup>9</sup>	234	[80]
Oxygen	6	2.486	100.019	10000	1.035×10 <sup>9</sup>	76	[81]
Neon	4	0.895	15.026	1000	1.035×10 <sup>9</sup>	67	[82]
Sodium	9	1.933	11.600	1000	1.293×10 <sup>8</sup>	193	[83, 84]
Magnesium	23	0.431	86.110	20000	5.174×10 <sup>8</sup>	371	[85]
Aluminum	15	0.270	32.383	10000	2.156×10 <sup>8</sup>	240	[86]
Silicon	7	2.329	18.632	50000	1.293×10 <sup>8</sup>	85	[87, 88]
LiF	8	2.082	15.701	10000	1.035×10 <sup>9</sup>	91	[89]
B <sub>4</sub> C	16	0.251	50.174	2000	5.174×10 <sup>8</sup>	291	[90]
BN	16	0.226	45.161	2000	5.174×10 <sup>8</sup>	311	[91]
CH <sub>4</sub>	16	0.072	14.376	6736	1.293×10 <sup>8</sup>	247	[92, 93]
CH <sub>2</sub>	16	0.088	17.598	6736	1.293×10 <sup>8</sup>	248	[92, 93]
C <sub>2</sub> H <sub>3</sub>	16	0.097	19.389	6736	1.293×10 <sup>8</sup>	247	[92, 93]
CH	16	0.105	21.000	6736	1.293×10 <sup>8</sup>	248	[92, 93]
C <sub>2</sub> H	16	0.112	22.430	6736	1.293×10 <sup>8</sup>	245	[92, 93]
MgO	19	0.357	71.397	20000	5.174×10 <sup>8</sup>	286	[94]
MgSiO <sub>3</sub>	16	0.321	64.158	6736	5.174×10 <sup>8</sup>	284	[95, 96]

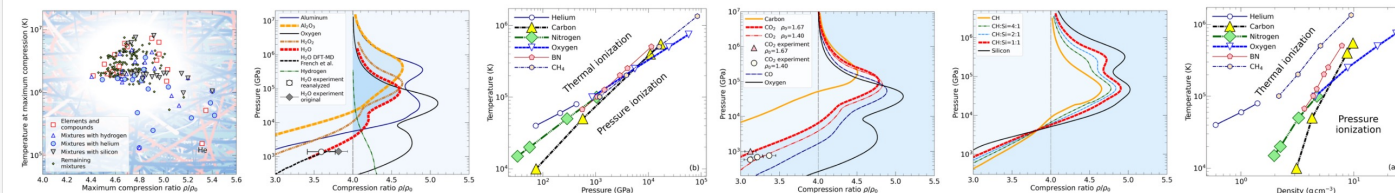
# First-Principles Equation of State Database online <http://militzer.berkeley.edu/FPEOS>

## First-Principles Equation of State (FPEOS) Database for Warm Dense Matter Computation

Authors: [B. Militzer](#), F. Gonzalez-Cataldo, S. Zhang, K. P. Driver, F. Soubiran

With the goal in mind of making WDM computations more reliable and efficient, we make available our EOS tables for 11 elements and 10 compounds as well as the C++ computer codes for their interpolation. Python code is provided to generate graphs of shock Hugoniot curve, isentropes, isobars, and isotherms for compounds and user-defined mixtures. We put together this first-principles equation of state (FPEOS) database for matter at extreme conditions by combining results from path integral Monte Carlo and density functional molecular dynamics simulations of the elements H, He, B, C, N, O, Ne, Na, Mg, Al and Si as well as the compounds LiF, B<sub>4</sub>C, BN, CH<sub>4</sub>, CH<sub>2</sub>, C<sub>2</sub>H<sub>3</sub>, CH, C<sub>2</sub>H, MgO, and MgSiO<sub>3</sub>. For all these materials, we provide the pressure and internal energy over a wide density-temperature range from  $\sim 0.5$  to 50 g/cc and from  $\sim 10^4$  to  $10^9$  K. This database encompasses the results from approximately 5000 different first-principles simulations. It allows one to compute isobars, adiabats, and shock Hugoniot curves in the regime of L and K shell ionization. Invoking the [linear mixing approximation](#), one can study the properties of user-defined mixtures at high density and temperature.

Recommended citation: B. Militzer, F. Gonzalez-Cataldo, S. Zhang, K. P. Driver, F. Soubiran, "[First-Principles Equation of State Database for Warm Dense Matter Computation](#)", *Physical Review E* **103** (2021) 013203.



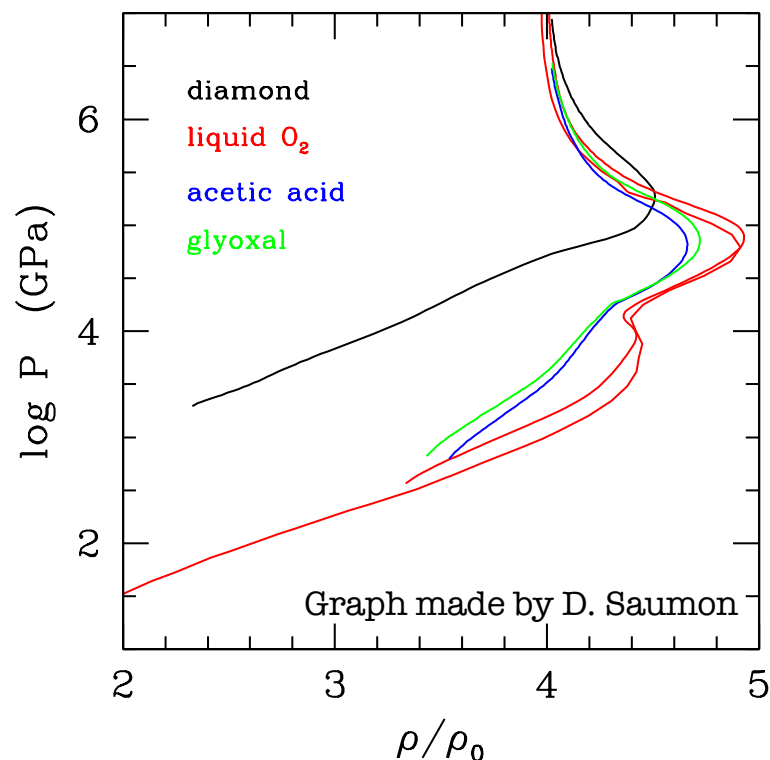
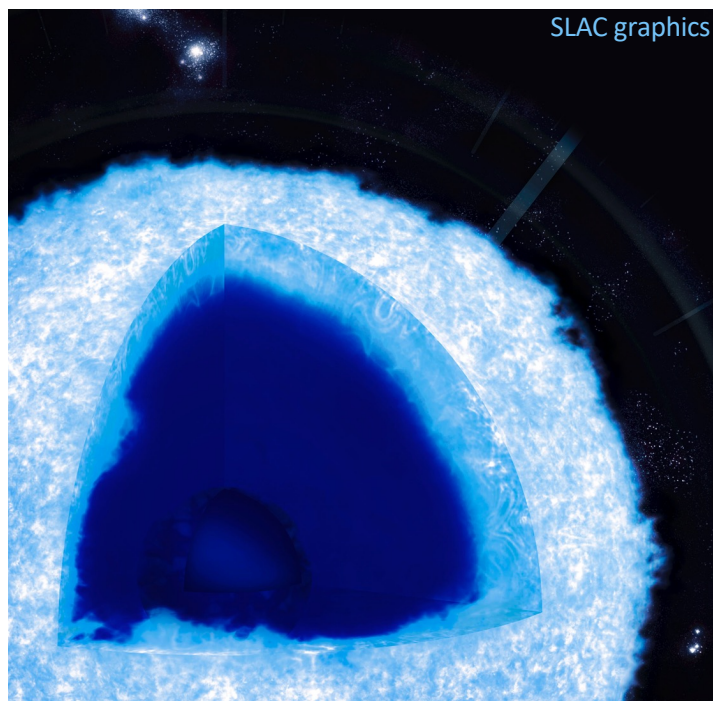
5000 first-principles calculations have been combined into our FPEOS database. So anyone can predict shock Hugoniot curves for a variety of compounds and mixtures. This will make warm dense matter calculations more reliable and efficient.



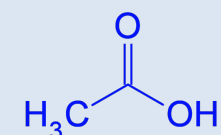
# NIF Gbar Experiment: Equations of State of C-O Mixtures in White Dwarf Stars

PI: D. Saumon (LANL), Blouin, Glenzer, Swift, Kritcher, Doppner, Whitley, Lazicki, Falcone, Militzer

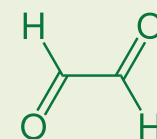
We propose to make EOS measurements along the Hugoniot with the Gbar platform of carbon-oxygen rich materials that resemble conditions in White Dwarf stars.



Acetic acid



Glyoxal



Glyoxal  $\text{C}_2\text{O}_2\text{H}_2$  comp:~/fpeos> fpeos binaryMixture EOS1=6 2.0 EOS2=18 2.0 rho0=1.27 E0=-227.8

Acetic acid  $\text{C}_2\text{O}_2\text{H}_4$  comp:~/fpeos> fpeos binaryMixture EOS1=6 2.0 EOS2=16 2.0 rho0=1.049 E0=-229.0

**FPEOS**

**demo**