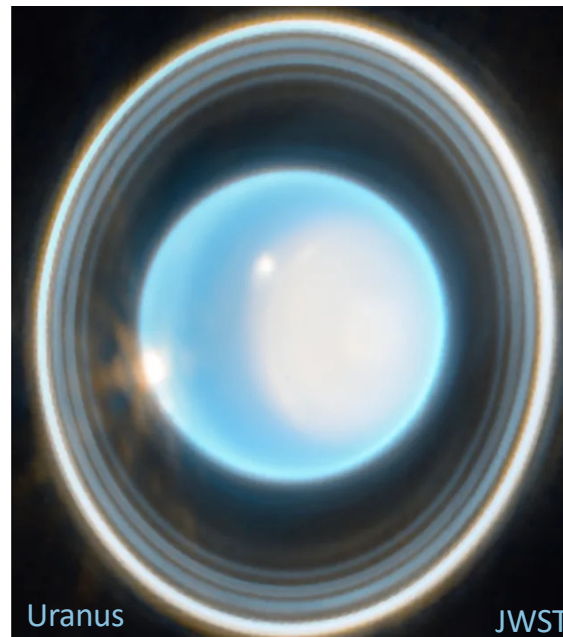
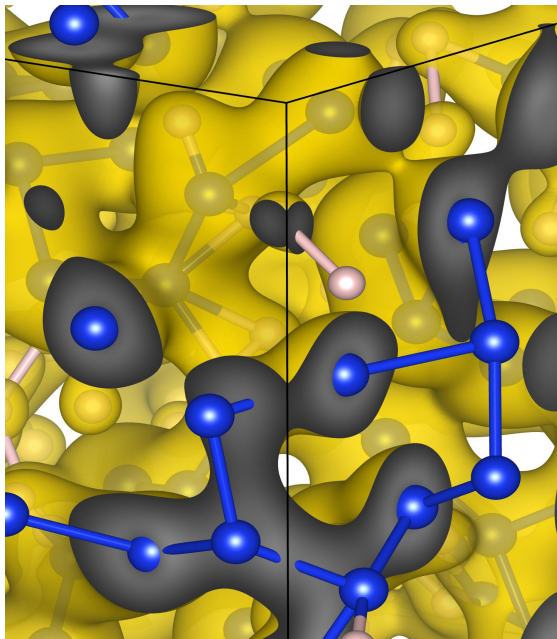


Interior Structure of Uranus and Neptune – Why Don't These Planets Generate Dipolar Magnetic Fields?



Burkhard Militzer

University of California, Berkeley

Big Questions that my Group Helps Address

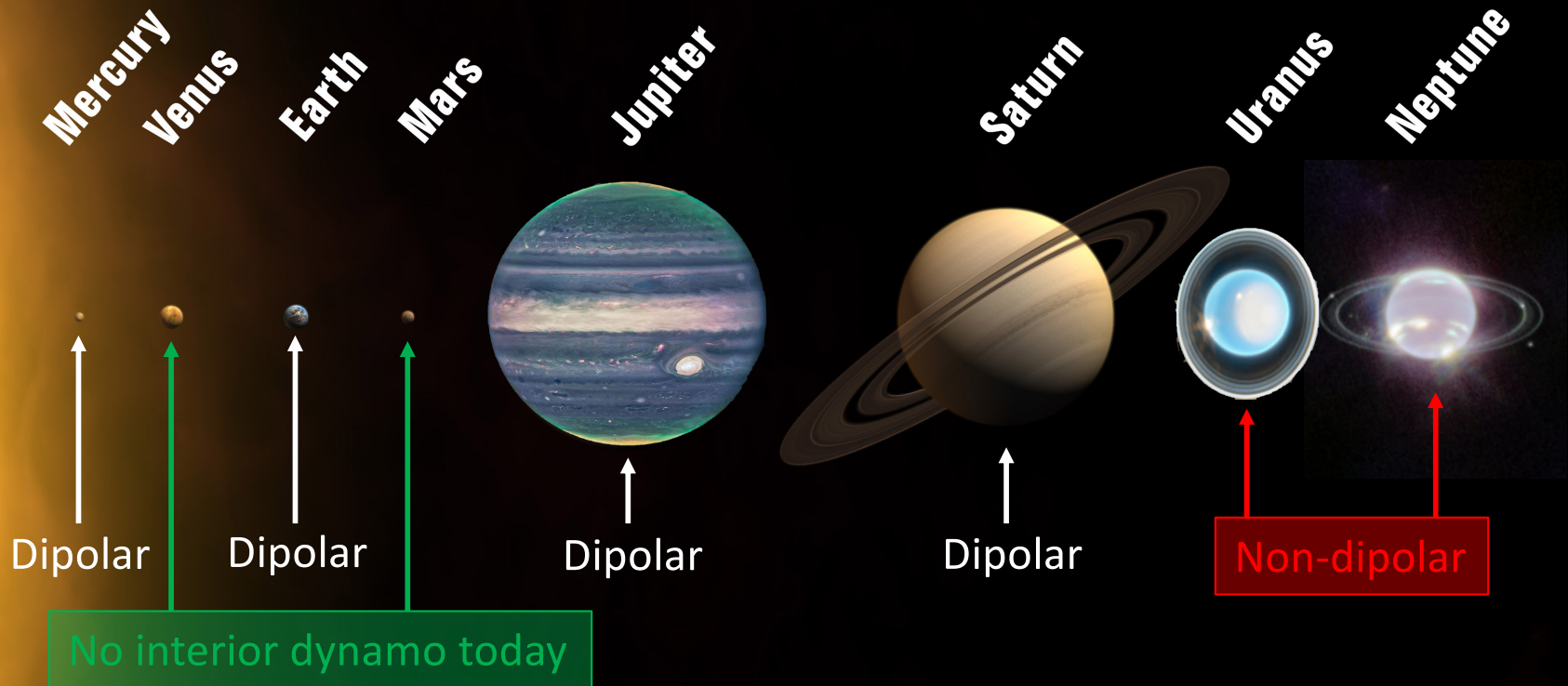
1. How did our solar system form?
2. What are giant planets made of?
3. How do materials behave at high pressure?



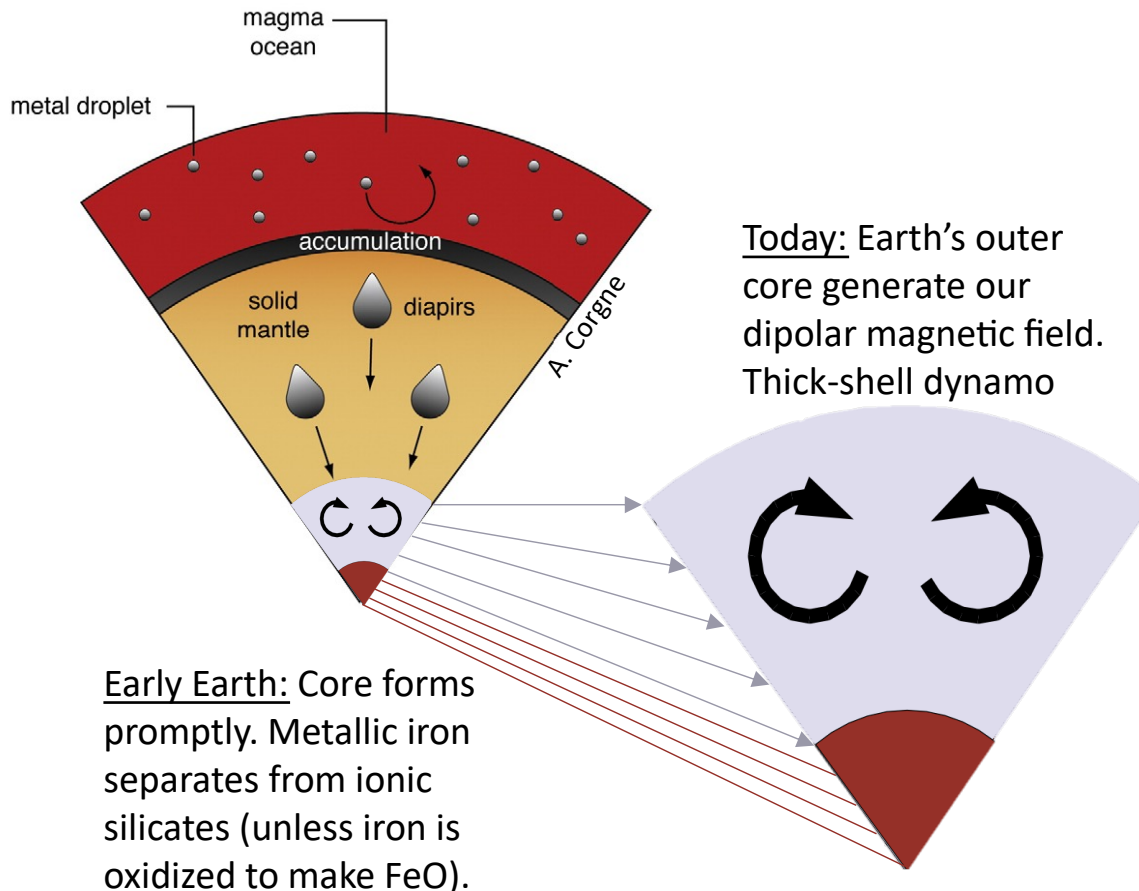
Supported by NSF through the Center for Matter at Atomic Pressure



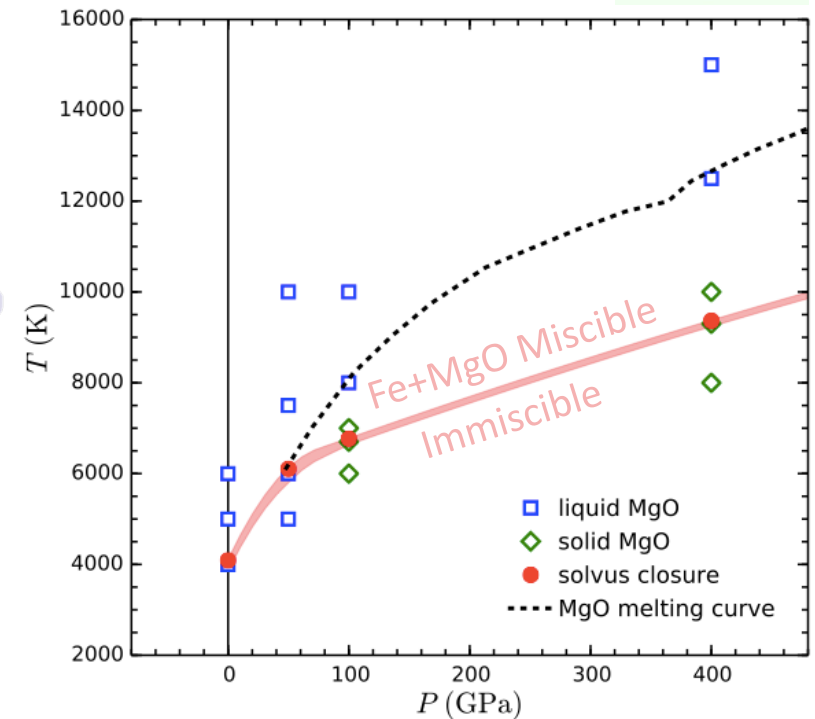
Magnetic Fields in our Solar System



Earth cores forms because iron and silicates are immiscible. Magnetic Field in Liquid Outer Core

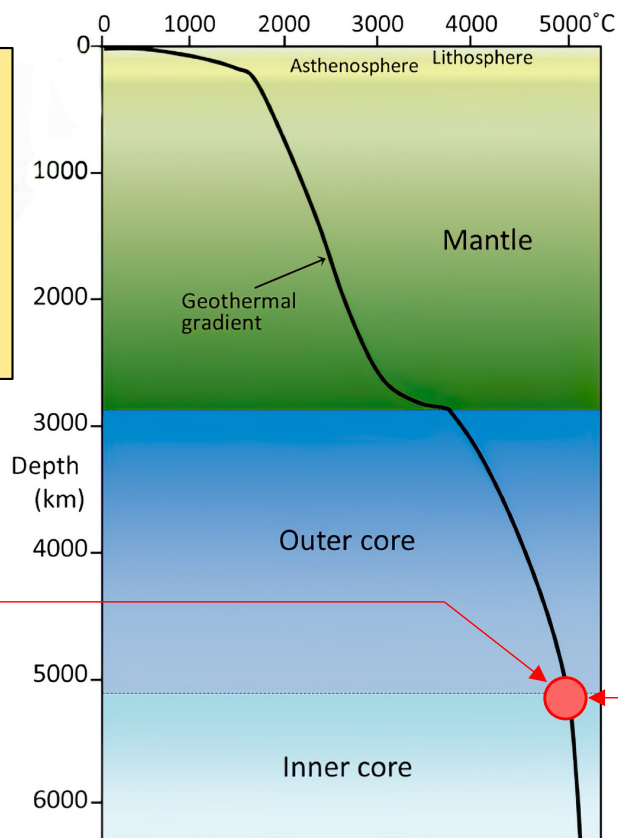
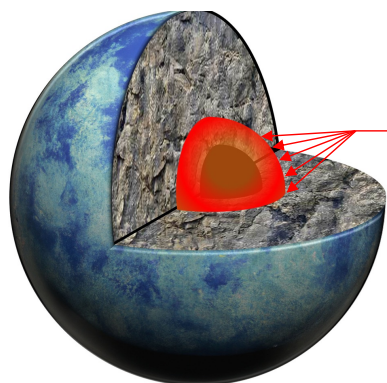


Wahl & BM (2015). At high temperature, elemental iron and MgO will mix.

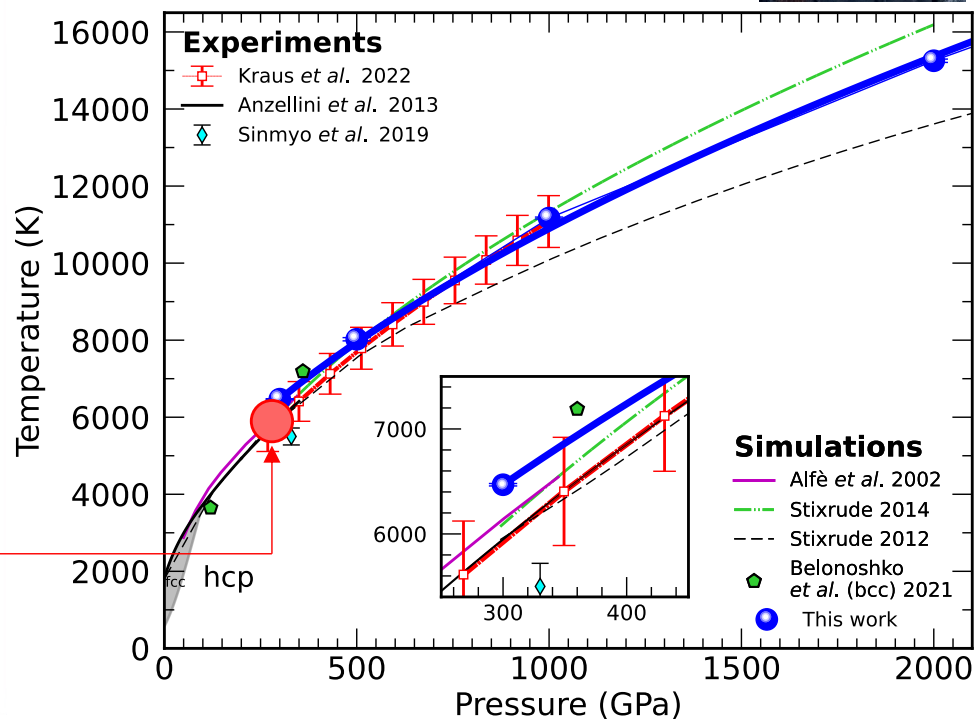


Earth has a solid inner core. So the Melting Temperature of Iron Constrains the Temperature of Earth's Core.

The Earth's core cannot be arbitrarily hot because otherwise there would be no solid inner core today.



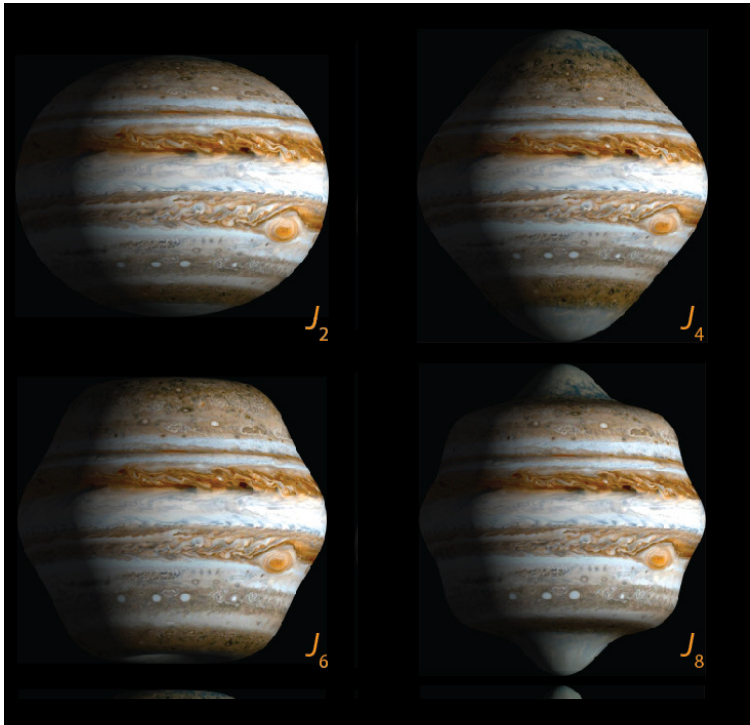
D. Alfe et al., Nature (2000)
F. Gonzalez, BM, Physical Review
Research 5 (2023) 033194



Gravity Field Measurements represented by harmonics J_n

Difference in Precision between single Flyby and Orbiting Mission

$$V(r, \mu) = \frac{GM}{r} \left[1 - \sum_{n=1}^{\infty} \left(\frac{a}{r} \right)^{2n} J_{2n} P_{2n}(\mu) \right]$$



Pioneer+Voyager Jupiter **flybys**

$$J_2 = 14697 \pm 1$$

$$J_4 = -584 \pm 5$$

$$J_6 = 31 \pm 20$$

Voyager Uranus **flyby**

$$J_2 = 3510 \pm 0.72$$

$$J_4 = -33.61 \pm 1$$

$$J_6 = ?$$

Measurements of Juno **orbiter**

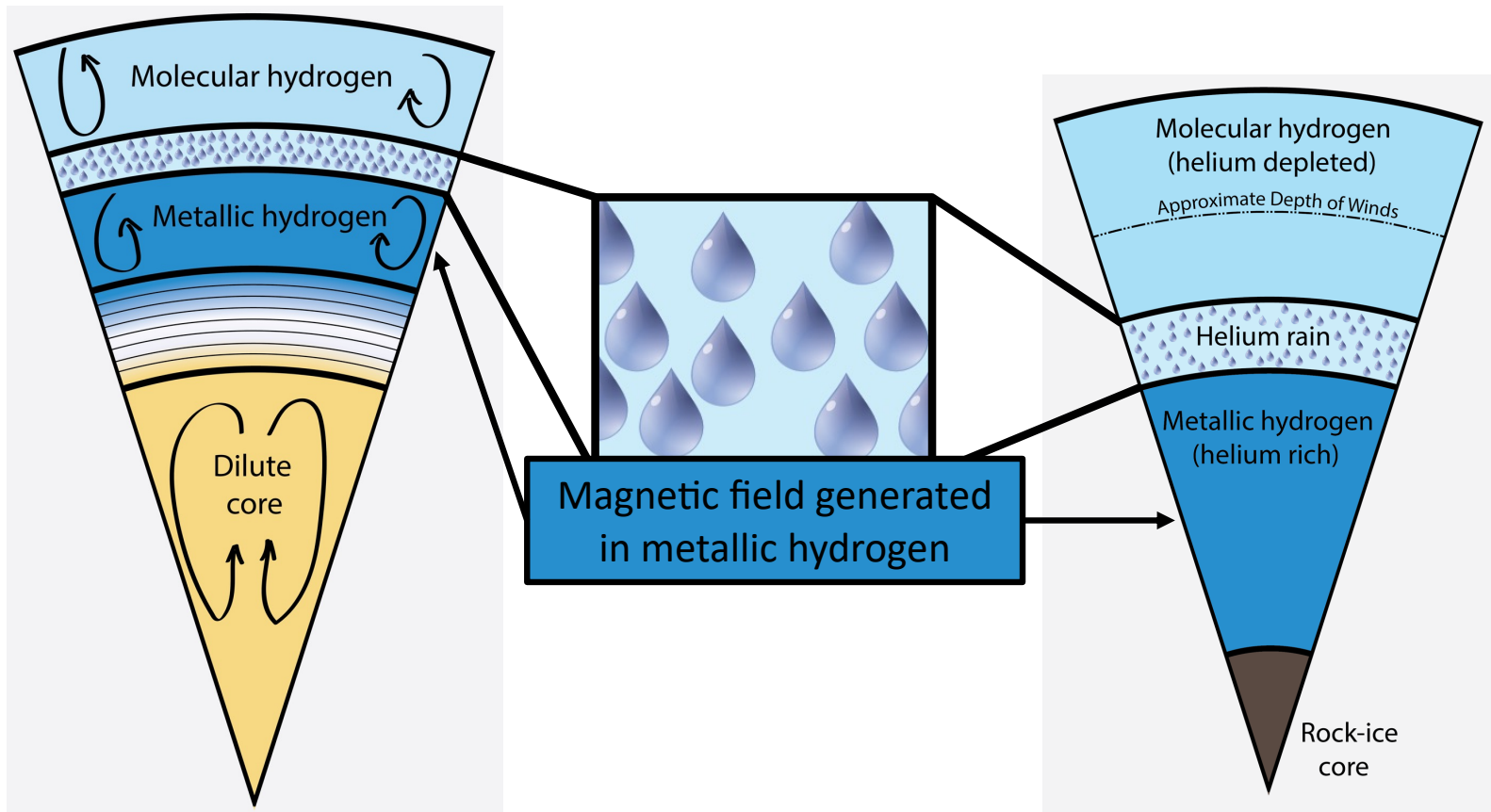
$$J_2 = 14696.5735 \pm 0.0017$$

$$J_4 = -586.6085 \pm 0.0024$$

$$J_6 = 34.2007 \pm 0.0067$$

Uranus **orbiter** would increase
gravity precision by factor 1000.

Models with Helium Rain for Jupiter and Saturn



Jupiter's interior with dilute core (Militzer et al., 2022)

Saturn's interior

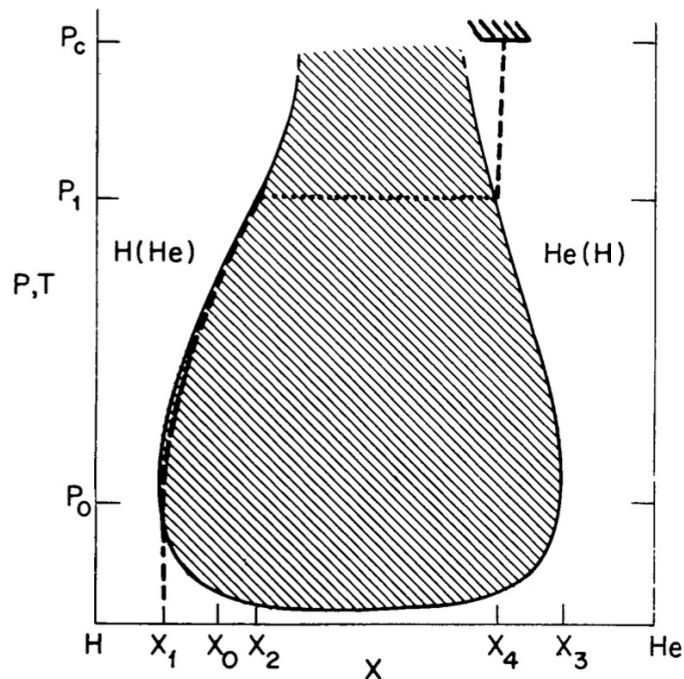
Stevenson & Salpeter (1977): Helium Rain Hypothesis to Explain Excess in Saturn's Thermal Emission

THE DYNAMICS AND HELIUM DISTRIBUTION IN HYDROGEN-HELIUM FLUID PLANETS

D. J. STEVENSON* AND E. E. SALPETER

Center for Radiophysics and Space Research and Physics Department, Cornell University

Received 1976 June 23; accepted 1977 April 13

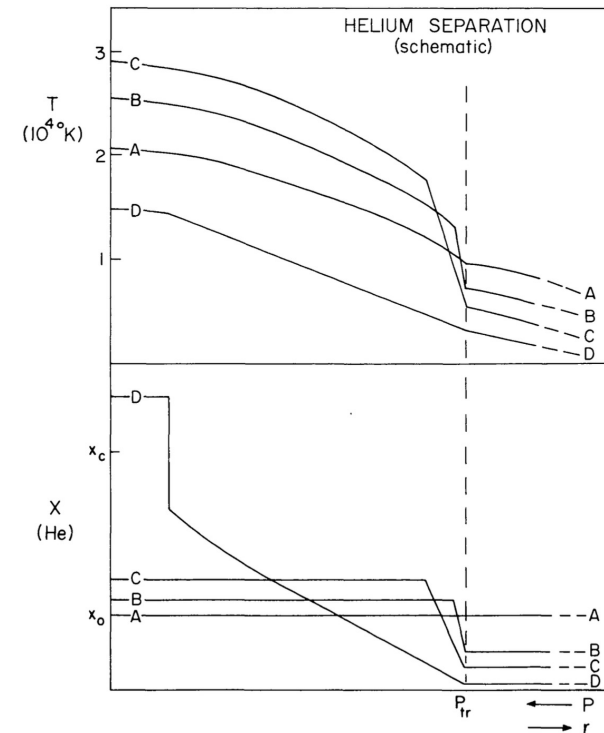


THE PHASE DIAGRAM AND TRANSPORT PROPERTIES FOR HYDROGEN-HELIUM FLUID PLANETS

D. J. STEVENSON AND E. E. SALPETER

Center for Radiophysics and Space Research and Physics Department, Cornell University

Received 1976 June 23; accepted 1977 April 13

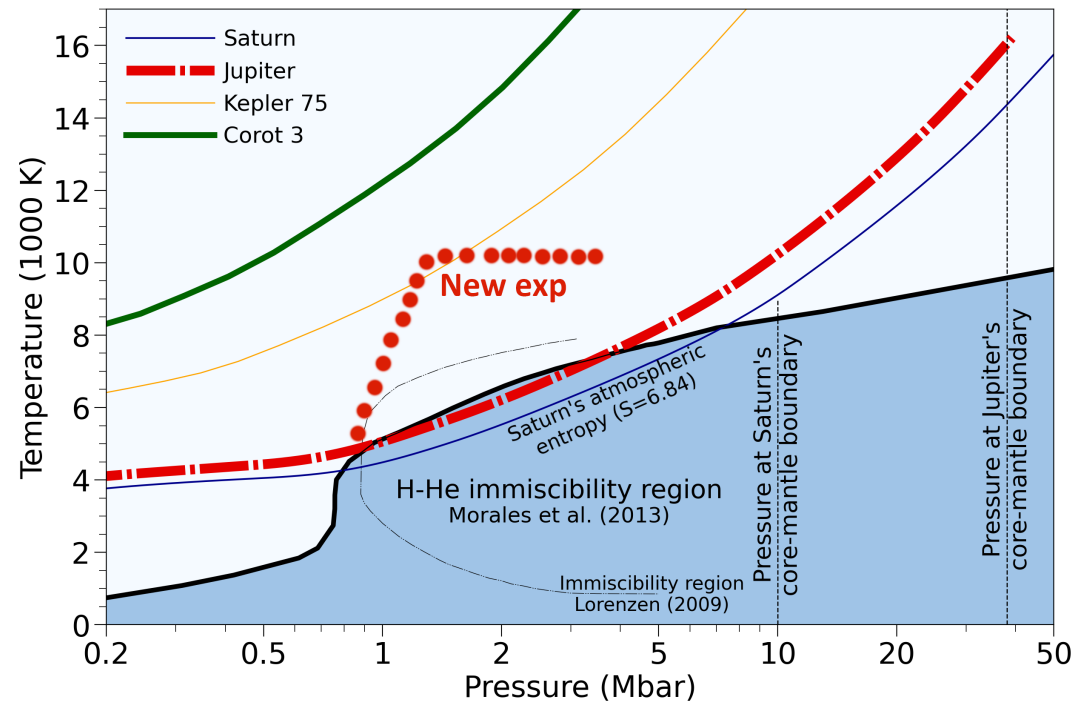
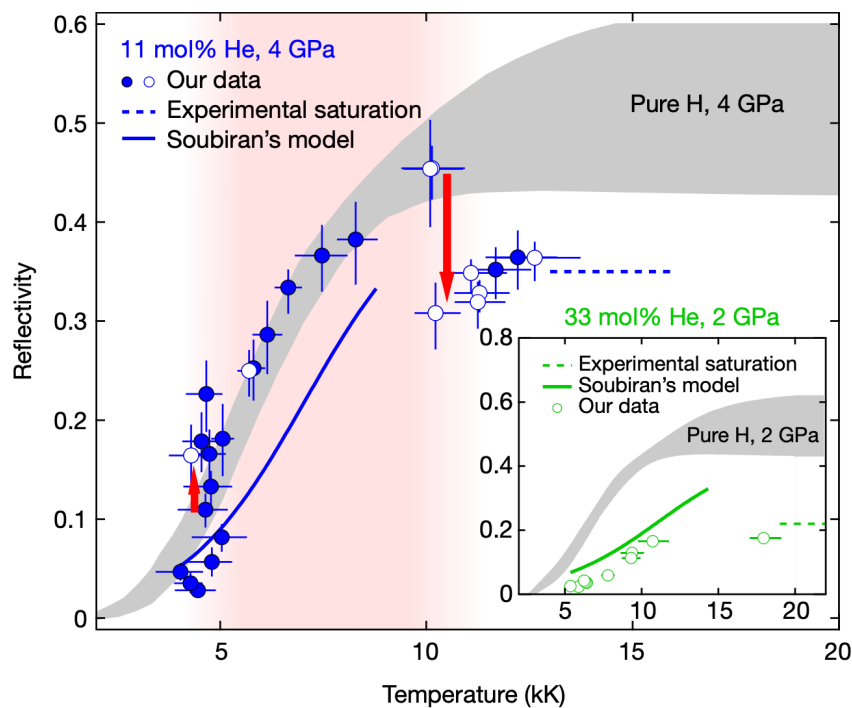


Evidence of hydrogen–helium immiscibility at Jupiter-interior conditions

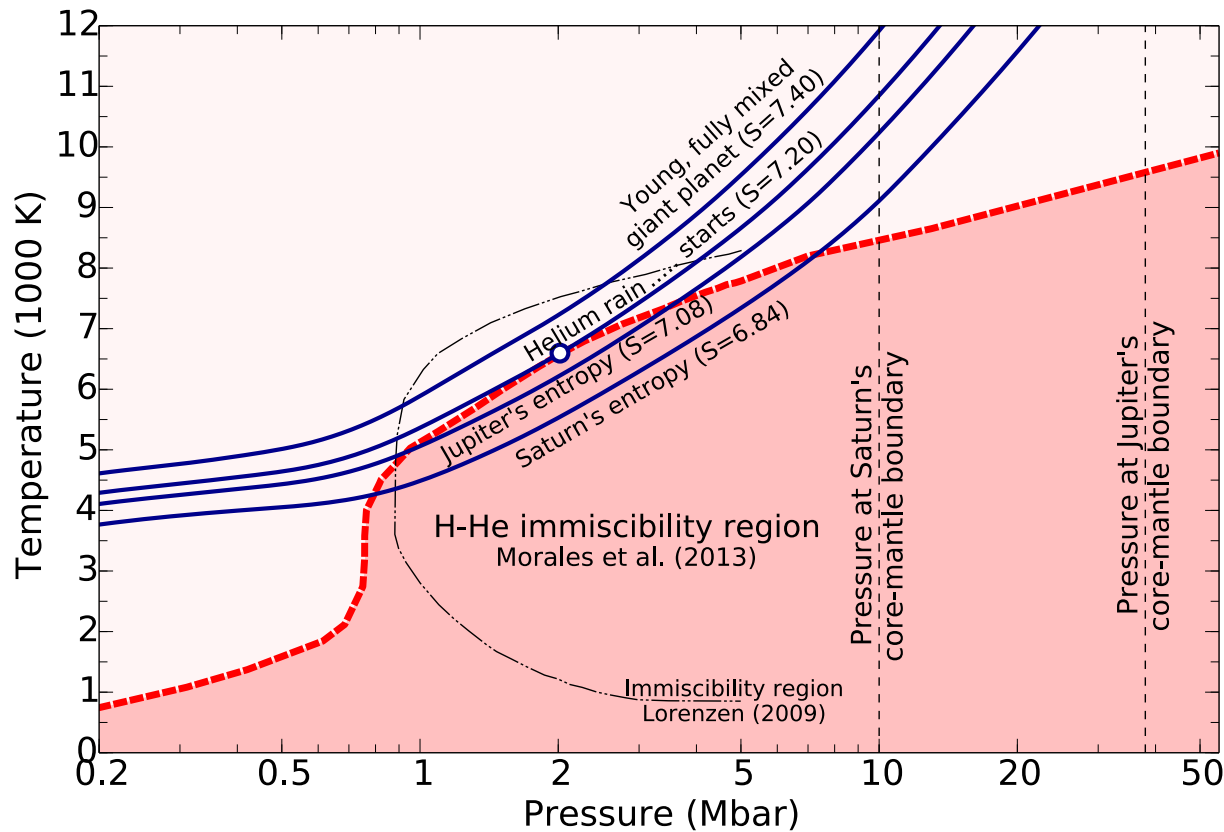
<https://doi.org/10.1038/s41586-021-03516-0>

S. Brygoo¹, P. Loubeyre¹, M. Millot², J. R. Rygg³, P. M. Celliers², J. H. Eggert², R. Jeanloz⁴ & G. W. Collins³

Received: 13 October 2015



Planets cool convectively: So we assume most of their interior layers are isentropic and homogeneous



One enjoyable way to observe convection: Ordering miso soup in a sushi restaurant

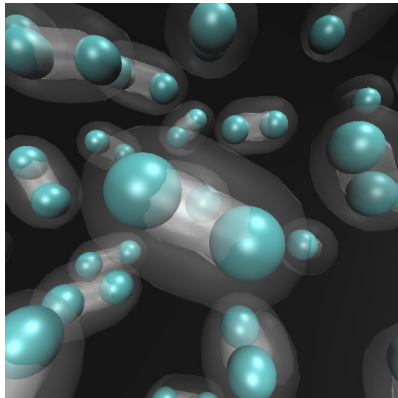
What are ab initio simulations?

Schrödinger equation:

$$-\frac{\hbar^2}{2m} \vec{\nabla}^2 \psi(\vec{r}) + V(\vec{r}) \psi(\vec{r}) = E \psi(\vec{r})$$

Look for an antisymmetric solution (Pauli exclusion):

$$\Psi(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \chi_1(\mathbf{x}_1) & \chi_2(\mathbf{x}_1) & \cdots & \chi_N(\mathbf{x}_1) \\ \chi_1(\mathbf{x}_2) & \chi_2(\mathbf{x}_2) & \cdots & \chi_N(\mathbf{x}_2) \\ \vdots & \vdots & & \vdots \\ \chi_1(\mathbf{x}_N) & \chi_2(\mathbf{x}_N) & \cdots & \chi_N(\mathbf{x}_N) \end{vmatrix}$$



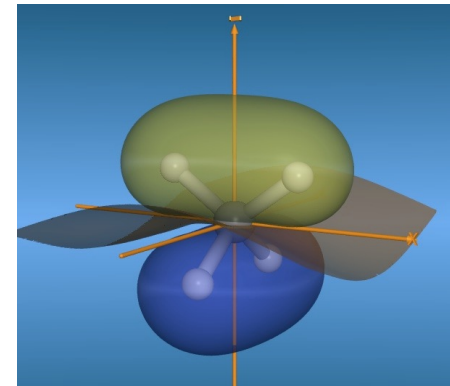
Simulation of molecular hydrogen

Density functional theory:

Generalized Gradient approximation (PBE)

Hybrid functionals (HSE for conductivity)

Quantum Monte Carlo



Methane - molecular orbitals

Calculate Free Energies and Entropy with Thermodynamic Integration. Here Applied to Molecular Hydrogen

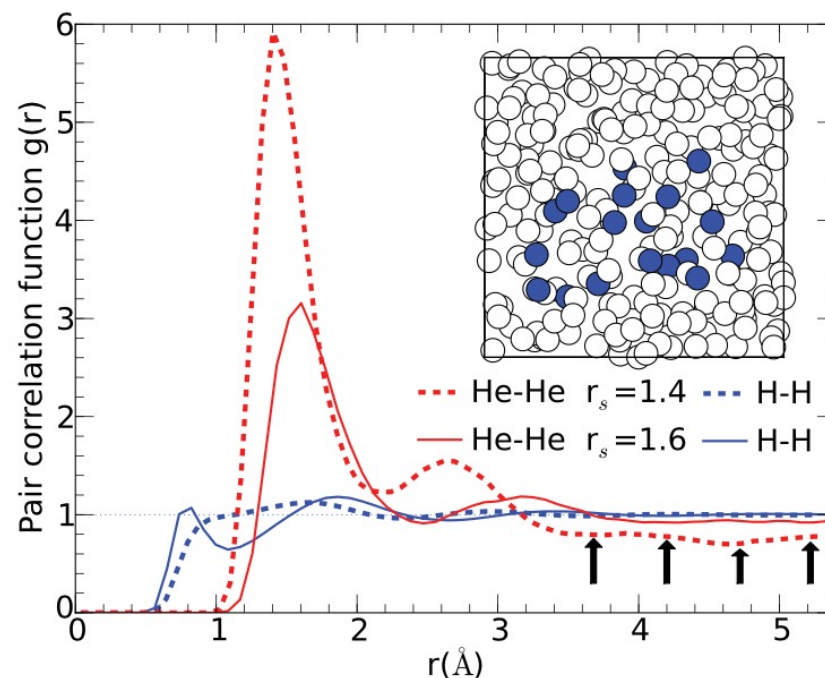
For fixed NVT

$$F_B - F_A = \int_0^1 d\lambda \langle U_B - U_A \rangle_\lambda$$

$$V_\lambda = \lambda V_{\text{KS}} + (1 - \lambda) V_{\text{cl}},$$

$$V = \sum_{i,j,i>j} g_{ij} V_{\text{mol}}(r_{ij}) + (1 - g_{ij}) V_{\text{inter}}(r_{ij})$$

Provides access to entropy and free energies that cannot be derived from standard MD simulations. For hydrogen, see BM, Phys. Rev. B **87** (2013) 014202.



Inside the immiscibility region, the simulations may spontaneously phase separate. Look for drop in $g(r)$ at large distances.

Determine Phase Transformations Either Dynamically or Thermodynamically

Perform MD simulations and wait for the system change to a new phase

- Heat until it melts
- Heat until it mixes
- Two-phase simulations
- Spontaneous phase separation

Gibbs free energy calculations

$$F_B - F_A = \int_0^1 d\lambda \langle U_B - U_A \rangle_\lambda$$

$$U(\lambda) = U_A + \lambda(U_B - U_A)$$

$$G_{DFT} = F_{DFT} + P_{DFT}V$$

Gibbs free energy (Which is more stable?)

$$TS_{DFT} = U_{DFT} - F_{DFT}$$

Entropy (ionic & electronic) (Construct isentropes)

Uranus

Mass = 14.5 Earths

Radius = 4.0 Earths

Density = 1.3 gram/cm³

Distance: 19.2 AU

Orbital Period: 84 years

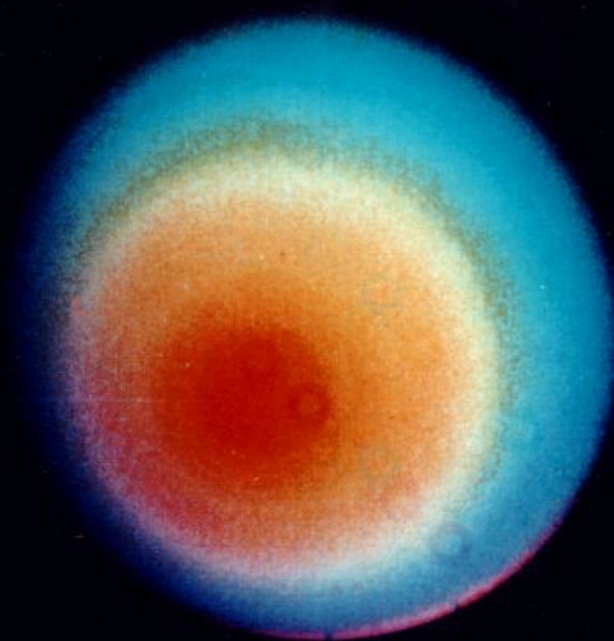
Rotation period: 17.2 hours.

Visible Light:

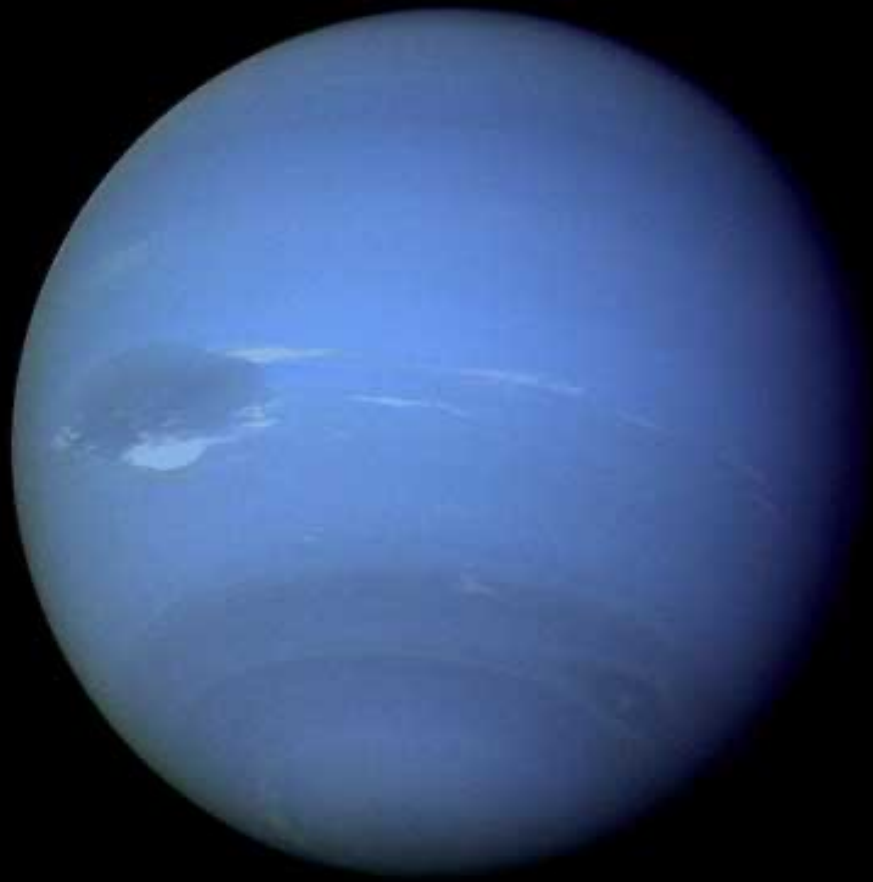


Featureless in visible light, because clouds are below haze layer of methane (colder than Saturn).

Infrared Light (almost no thermal emission):



Neptune



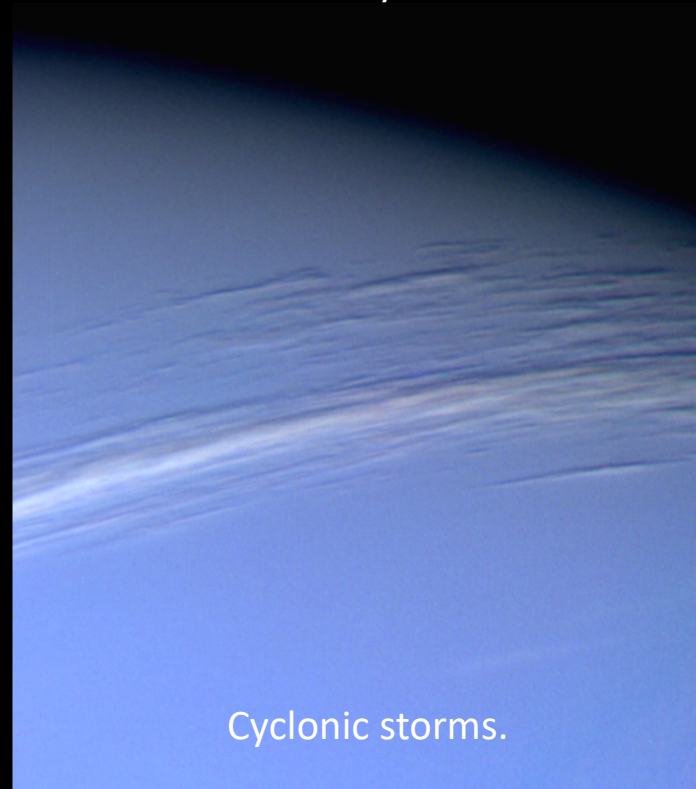
Mass = 17 Earths

Radius = 3.9 Earths

Density = 1.76 x water

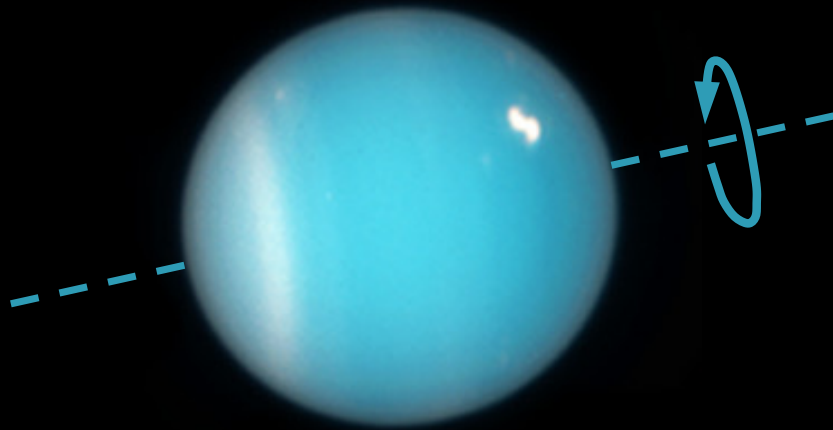
Distance: 30 AU

Orbital Period: 163 years



Cyclonic storms.

Uranus



Neptune



Open questions:

- Tilt of Uranus' rotation axis (giant impact hypothesis)
- Different CH₄ abundances (Uranus: 2.3 %, Neptune 1.4%)
- Uranus has almost no intrinsic heat flux (thermal boundary layer?)
- Uranus has a regular set of satellites. Neptune has only two in inclined orbits. (Triton is almost planet-like.)
- Interior composition uncertain ("Ice giants" misnomer)
- Unusual magnetic fields

Disclaimer: Due to lack of time, the following innovative papers cannot be discussed in this talk

- M Podolak, A Weizman, M Marley, Comparative models of Uranus and Neptune. *Planet. Space Sci.* 43, 1517–1522 (1995).
- K Soderlund, M Heimpel, E King, J Aurnou, Turbulent models of ice giant internal dynamics: Dynamos, heat transfer, and zonal flows. *Icarus* 224, 97–113 (2013).
- R Helled, P Bodenheimer, The formation of Uranus and Neptune: Challenges and implications for intermediate-mass exoplanets. *The Astrophys. J.* 789, 69 (2014).
- K Soderlund, S Stanley, The underexplored frontier of ice giant dynamos. *Philos. Trans. Royal Soc. A* 378, 20190479 (2020).
- E Bailey, DJ Stevenson, Thermodynamically governed interior models of Uranus and Neptune. *The Planet. Sci. J.* 2, 64 (2021).
- L Stixrude, S Baroni, F Grasselli, Thermal and tidal evolution of Uranus with a growing frozen core. *The Planet. Sci. J.* 2, 222 (2021).
- N Movshovitz, JJ Fortney, The promise and limitations of precision gravity: Application to the interior structure of Uranus and Neptune. *The Planet. Sci. J.* 3, 88 (2022).

1986: Voyager 2 arrives at Uranus and finds it has **no strong dipolar field**. Why might that be?

Podolak, Hubbard, Stevenson write in “Uranus” edited by Bergstrahl et al.

The most obvious and most popular explanation of the unusual field geometry is that we arrived at Uranus during a reversal. Indeed, the observed field geometry has some similarities to that inferred for Earth during geomagnetic reversals: a tilted dipole and an unusually large quadrupole (Merrill and McElhinny 1983). The problem with this explanation is that if Uranus is similar to Earth, then the probability of encountering the planet during a reversal event is only about 1% (the Earth’s field takes a few thousand years to reverse, yet the time between reversals is very long, typically a few hundred thousand years). If we accept the reversal explanation, then we must either accept an improbable chance or say that Uranus differs from the Earth in some very substantial way. The latter explanation seems attractive but dif-

1986: Voyager 2 arrives at Uranus and finds it has **no strong dipolar field**. Why might that be?

1989: Voyager 2 arrives at Neptune and determined that planet does not have strong dipole field either

Stevenson write in "Uranus" edited by Bergstrahl et al. The most popular explanation of the unusual field geometry is a reversal. Indeed, the observed field geometry has some similarities to Earth during geomagnetic reversals: a tilted dipole axis (Merrill and McElhinny 1983). The problem with this explanation is that if the field is similar to Earth, then the probability of encountering a reversal event is only about 1% (the Earth's field takes a few hundred thousand years to reverse, yet the time between reversals is very long, typically a few hundred thousand years). If we accept the reversal explanation, then we must either accept an improbable chance or say that Uranus differs from the Earth in some very substantial way. The latter explanation seems attractive but dif-

Ruzmaikin & Starchenko (1991): U+N generate their magnetic fields in a THIN SHELL

On the Origin of Uranus and Neptune Magnetic Fields

A. A. RUZMAIKIN

Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation, Troitsk, Moscow Region, USSR

AND

S. V. STARCHENKO

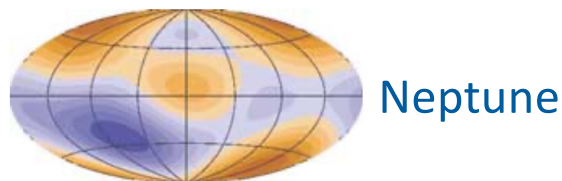
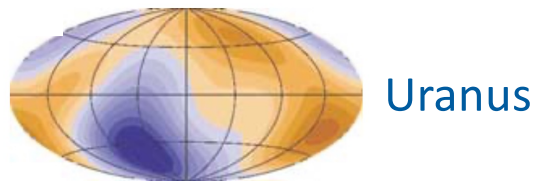
Schmidt Institute of Physics of the Earth, Moscow, USSR

Received July 6, 1990; revised April 9, 1991

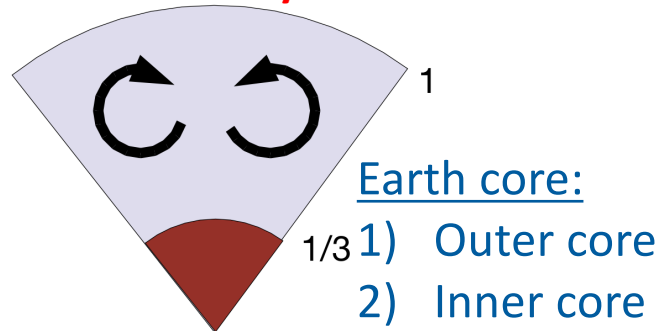
The Uranus and Neptune magnetic fields discovered by Voyager 2 can be explained by a dynamo acting in a thin conductive convective shell existing at the bottom of the icy oceans of the planets. The main helicity and differential rotation are the source for the dynamo which effectively excites nonaxisymmetric modes of the mean magnetic field. Estimates of the magnetic field amplitude in the nonlinear regime and of the inclination between the magnetic moment and the rotation axis are given. © 1991 Academic Press, Inc.

In this paper a model for the generation of the mean magnetic fields of Uranus and Neptune by action of the mean helicity of the convective motions and differential rotation is constructed. The field is generated in a thin shell where the conditions for self-excitation and the rates of growth for the axisymmetric and nonaxisymmetric modes are closed. One result in particular is that axisymmetric and nonaxisymmetric components of the dipole magnetic field are of the same order. The sum of these

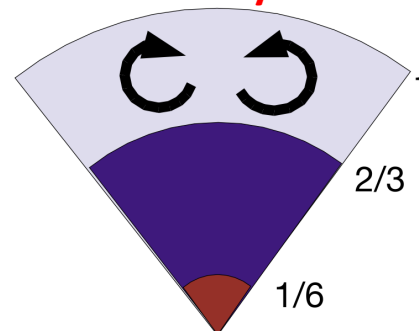
Stanley & Bloxham (2004): Numerical Simulations of Thin-Shell Dynamos matched Observed Fields



Thick-shell dynamo

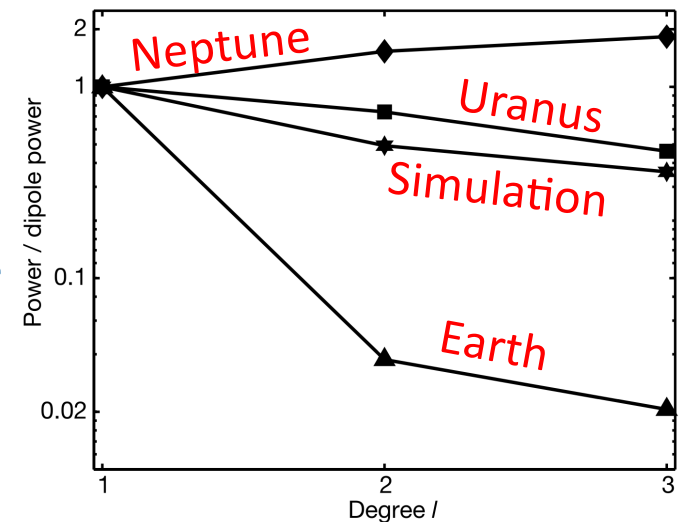


Thin-shell dynamo

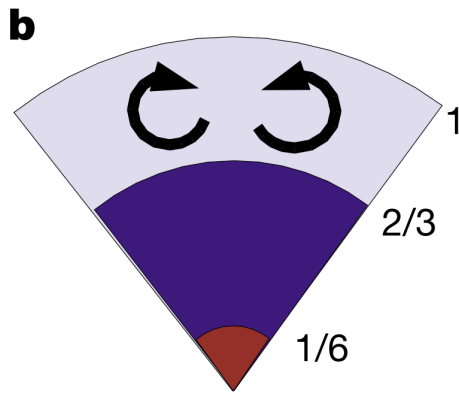


U+N model (Outer hydrogen layer excluded)

- 1) Thin upper dynamo layer
- 2) Lower less dynamo-active layer



Stanley & Bloxham (2004 and 2006): Most preferred Interior Structure Model

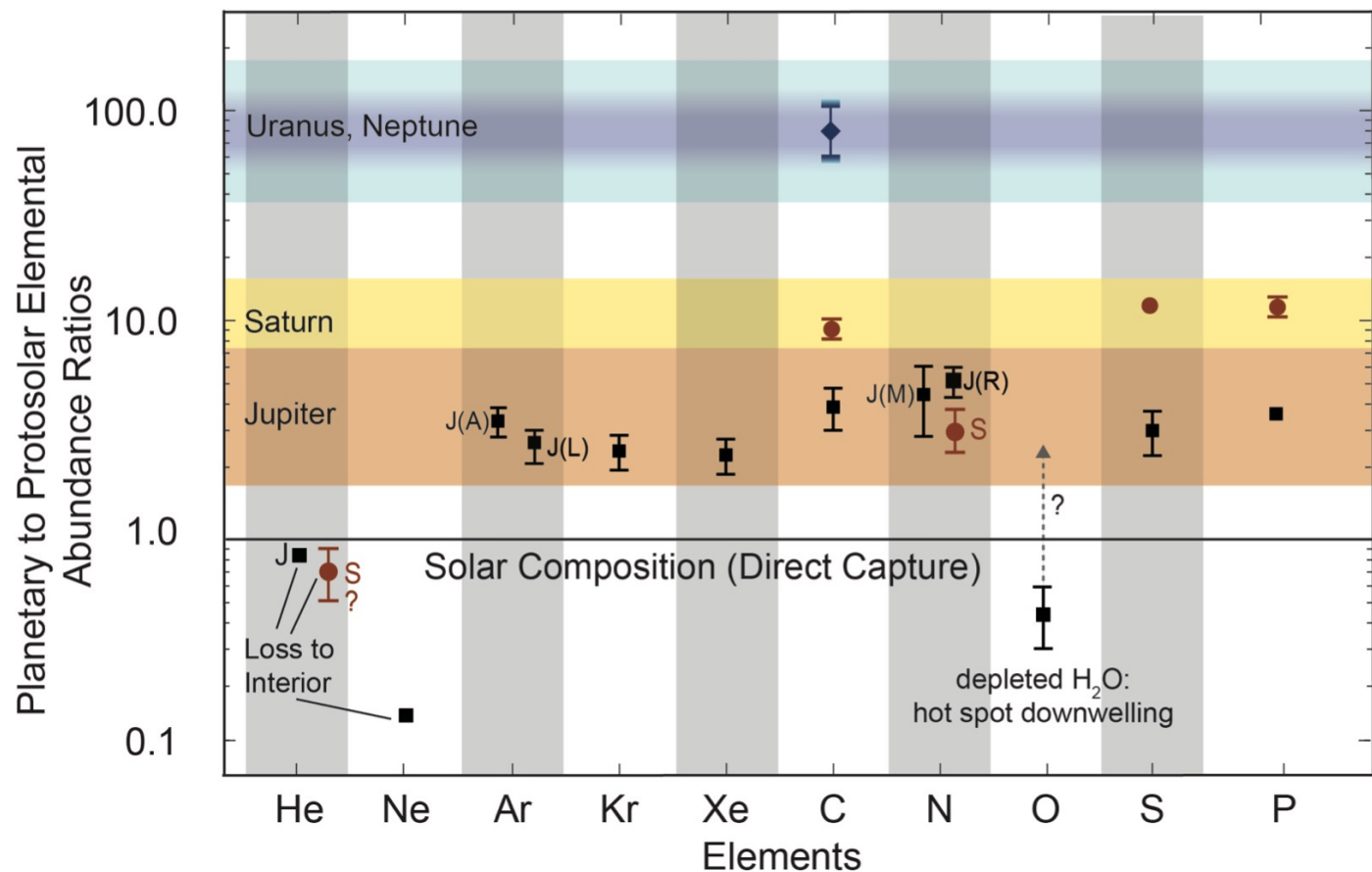


- ☐ U+N's magnetic fields are primarily generated in a thin outer layer
- ☐ This layer a homogeneous, electrically conducting fluid
- ☐ The inner-outer radius boundary is approximately at $2/3$.
- ☐ The inner layer is non-convecting, electrically conducting fluid
- ☐ They prefer a stably stratified electrically conducting fluid but a non-convecting, conducting solid might also work.

Open questions:

- 1) What is the composition of the upper layer?
- 2) What is the composition of the lower layer?
- 3) Why is the lower layer not convecting?

Atmospheres of Uranus and Neptune are Rich in Carbon (CH₄ was detected)



U+N: 40...100 × solar

Saturn: 9 × solar

Jupiter: 4 × solar

Atreya et al. (2016)
 Sromovsky et al. (2011)
 Voyager occultation,
 CH₄ clouds

M. Ross in Nature in 1981: Diamond Rain in U+N

The ice layer in Uranus and Neptune—diamonds in the sky?

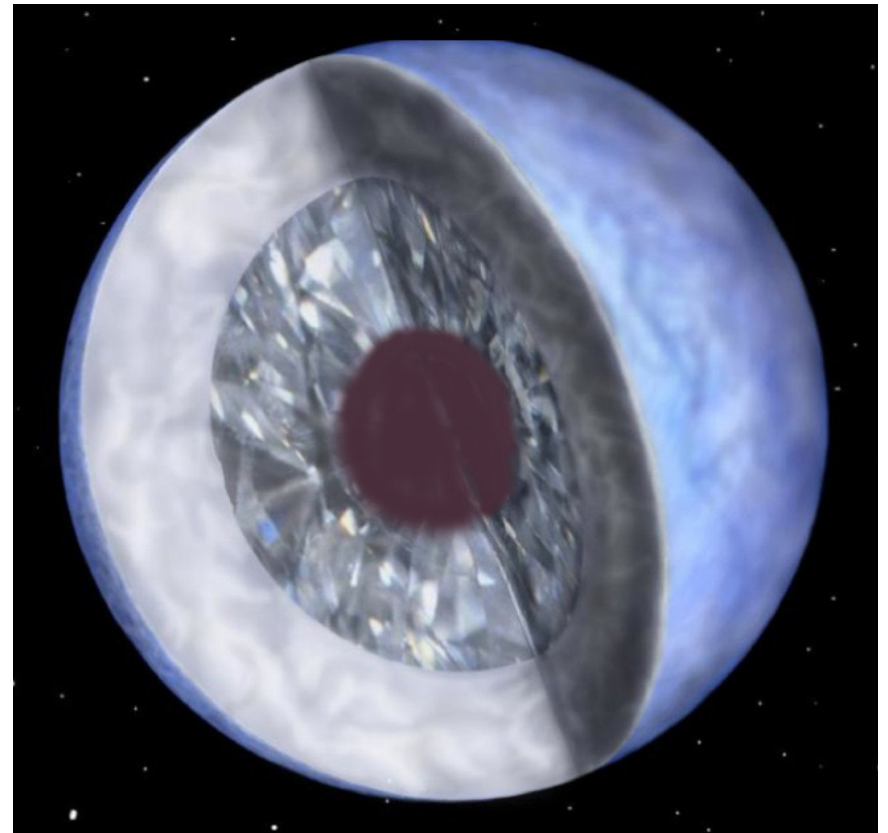
Marvin Ross

University of California, Lawrence Livermore National Laboratory,
Livermore, California 94550, USA

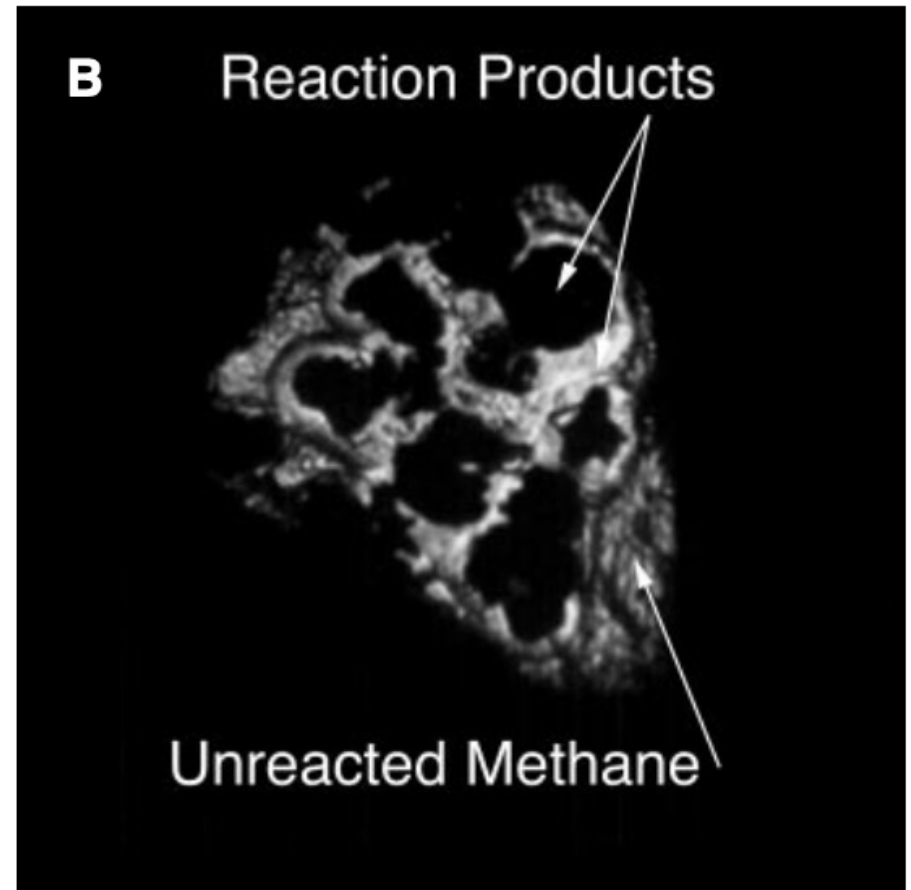
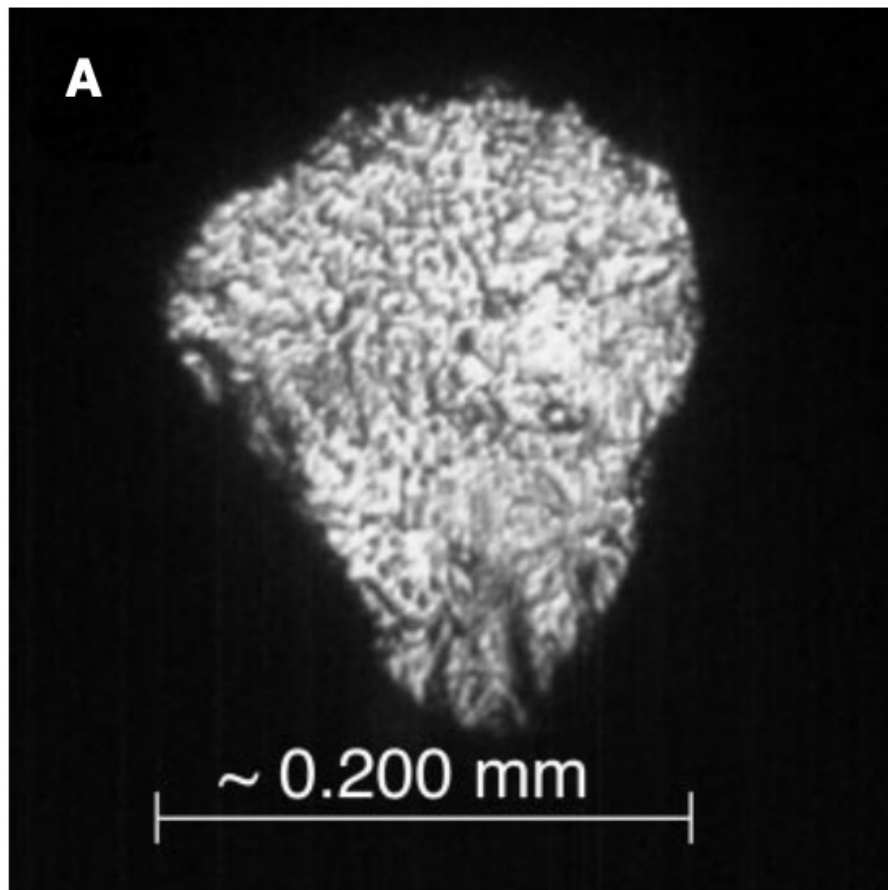
Many of the current models of Uranus and Neptune postulate a three-layer structure, consisting of an inner rocky core, a middle 'ice' layer of fluid, H_2O , CH_4 , NH_3 and an outer hydrogen-helium layer of solar composition¹. The estimated pressures and temperatures of the ice layer ranges from about 6 Mbar and 7,000 K at the inner core-ice boundary, to ~0.2 Mbar and 2,200 K at the outer ice/hydrogen-helium boundary. I point out here that shockwave experiments on these liquids²⁻⁵, as well as theoretical studies⁵⁻⁷, imply that the H_2O and NH_3 in the ice layer are almost totally ionized and the CH_4 has been pyrolysed to carbon, possibly in the metallic or diamond form^{8,9}.

In recent years shock-wave experiments have been carried

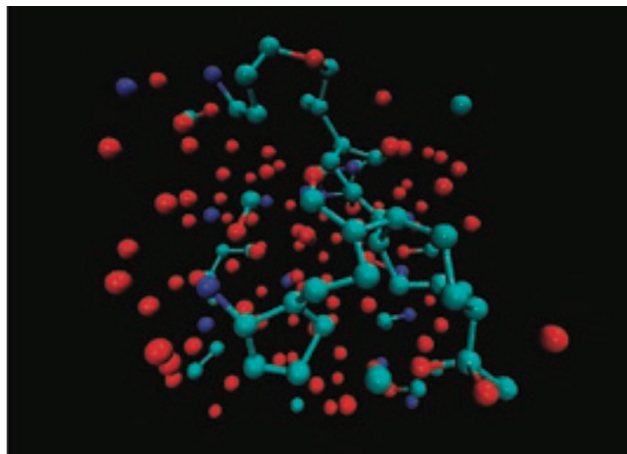
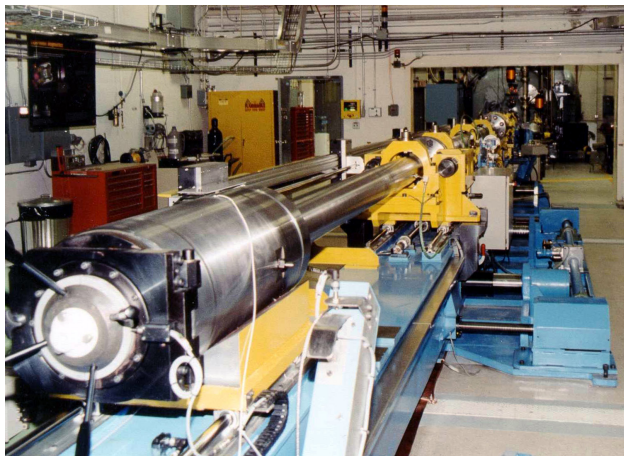
conclusions were based on chemical equilibria calculations which predicted that above 0.20 Mbar and 2,000 K, methane is converted into elemental carbon and molecular hydrogen. Recent theoretical calculations on Hugoniot of many hydro-



Benedetti et al. (1999): Laser Heated CH_4 forms
Diamonds at 10-50 GPa and 2000-3000 K



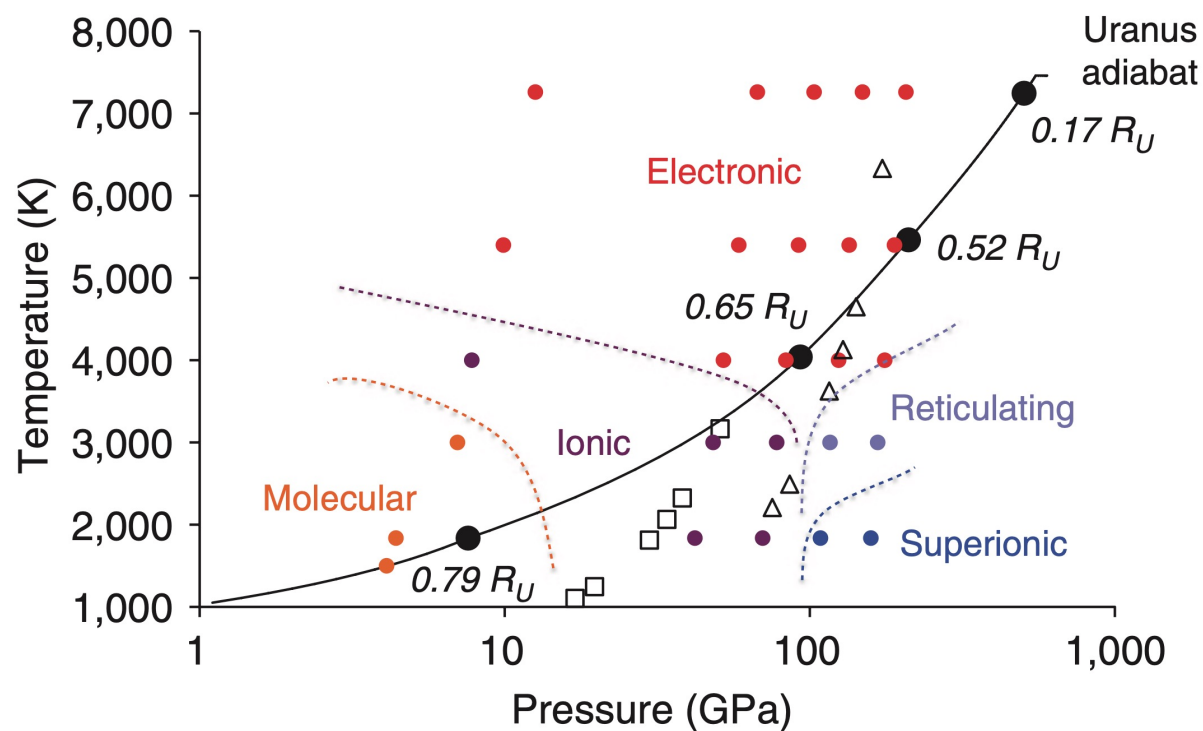
Experimental and Theoretical Work on “Synthetic Uranus” Mixture of H:O:C:N = 28:7:4:1



Received 15 Apr 2010 | Accepted 19 Jan 2011 | Published 22 Feb 2011

DOI: 10.1038/ncomms1198

Chemical processes in the deep interior of Uranus



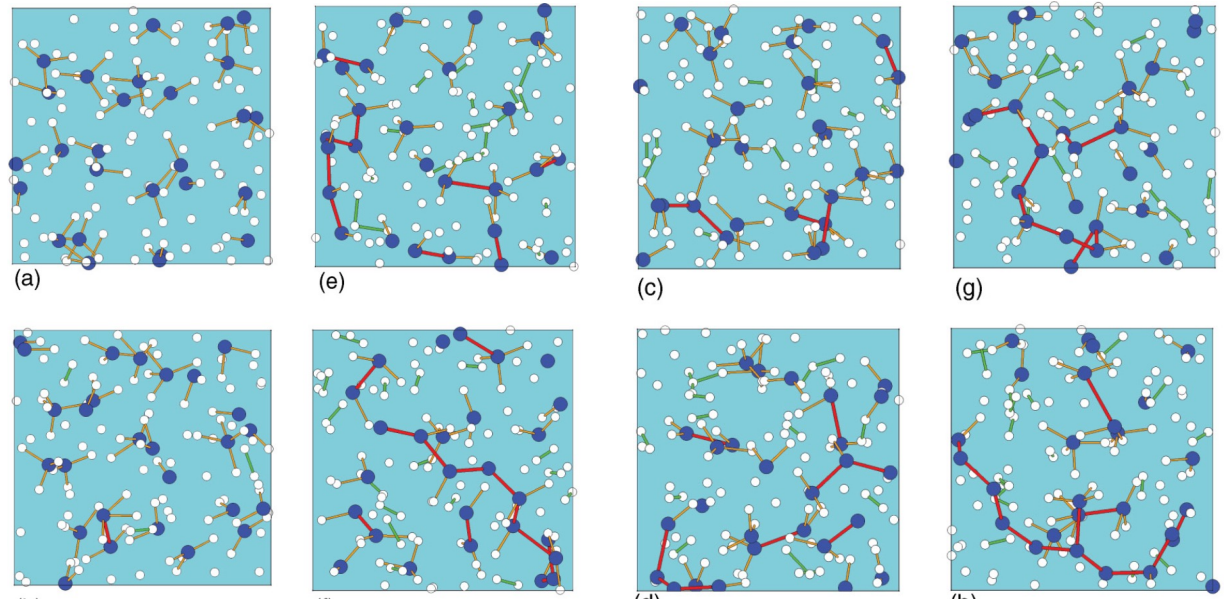
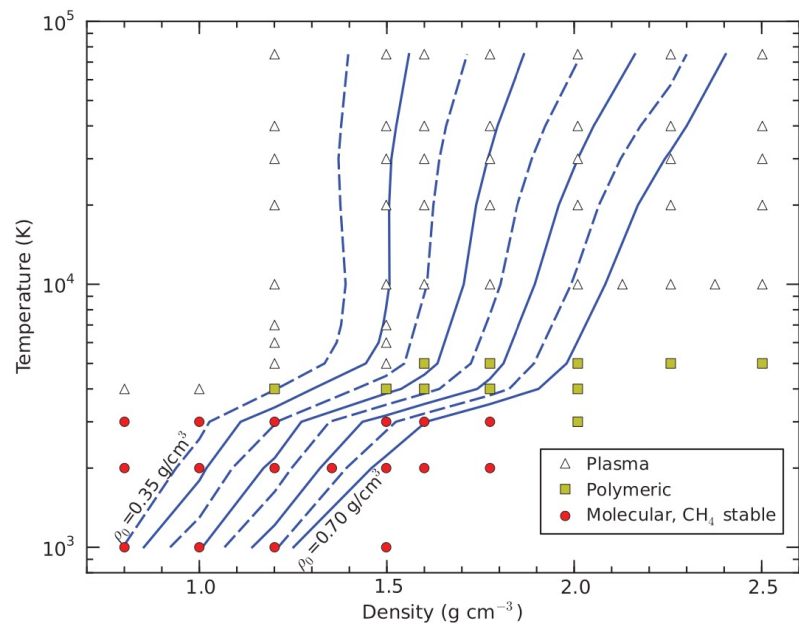
Chau, Hamel, Nellis (2011)

Our paper on polymeric state of methane under shock conditions (2012)

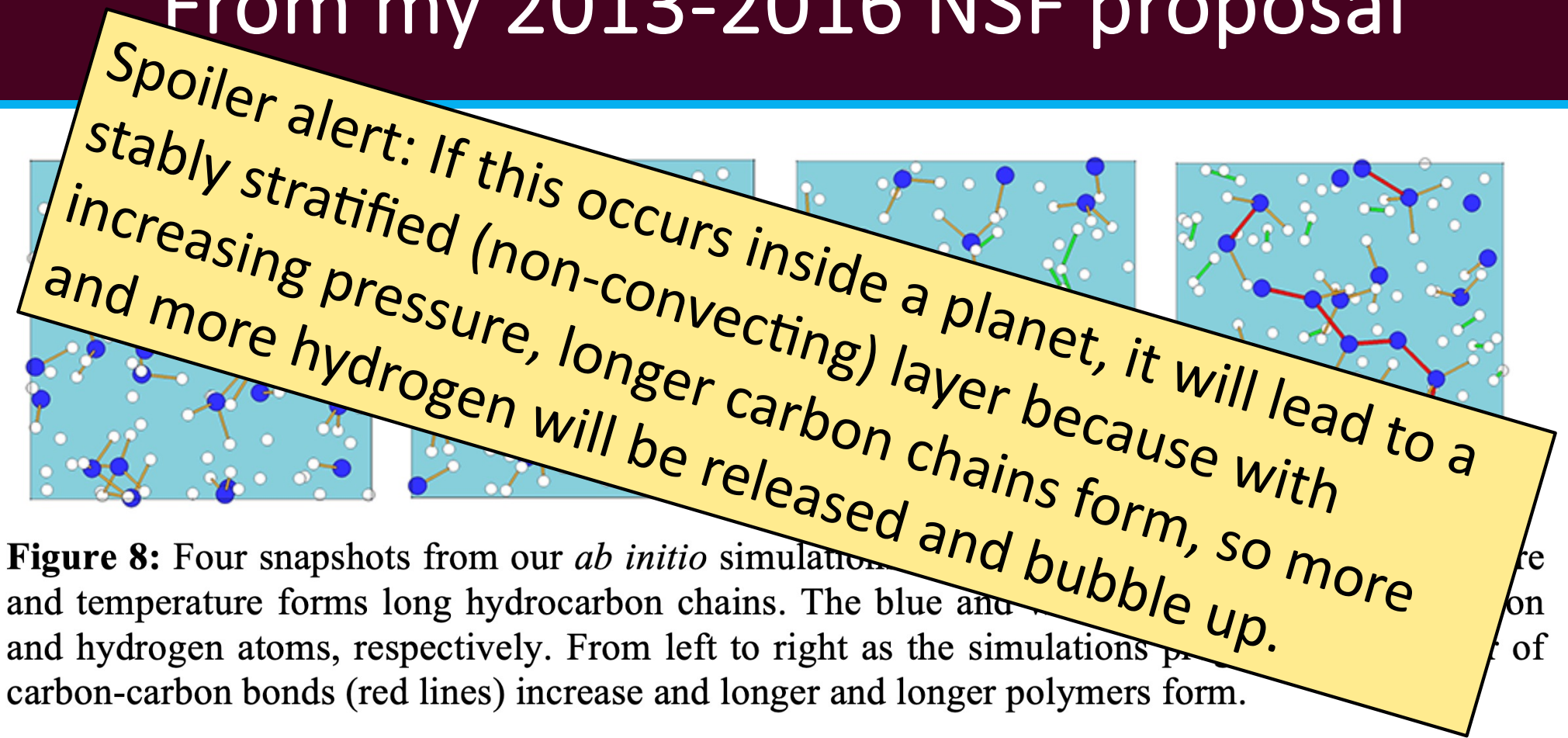
PHYSICAL REVIEW B **86**, 224113 (2012)

Ab initio simulations of hot dense methane during shock experiments

Benjamin L. Sherman,¹ Hugh F. Wilson,² Dayanthie Weeraratne,¹ and Burkhard Militzer^{2,3}



From my 2013-2016 NSF proposal



From my 2013-2016 NSF proposal

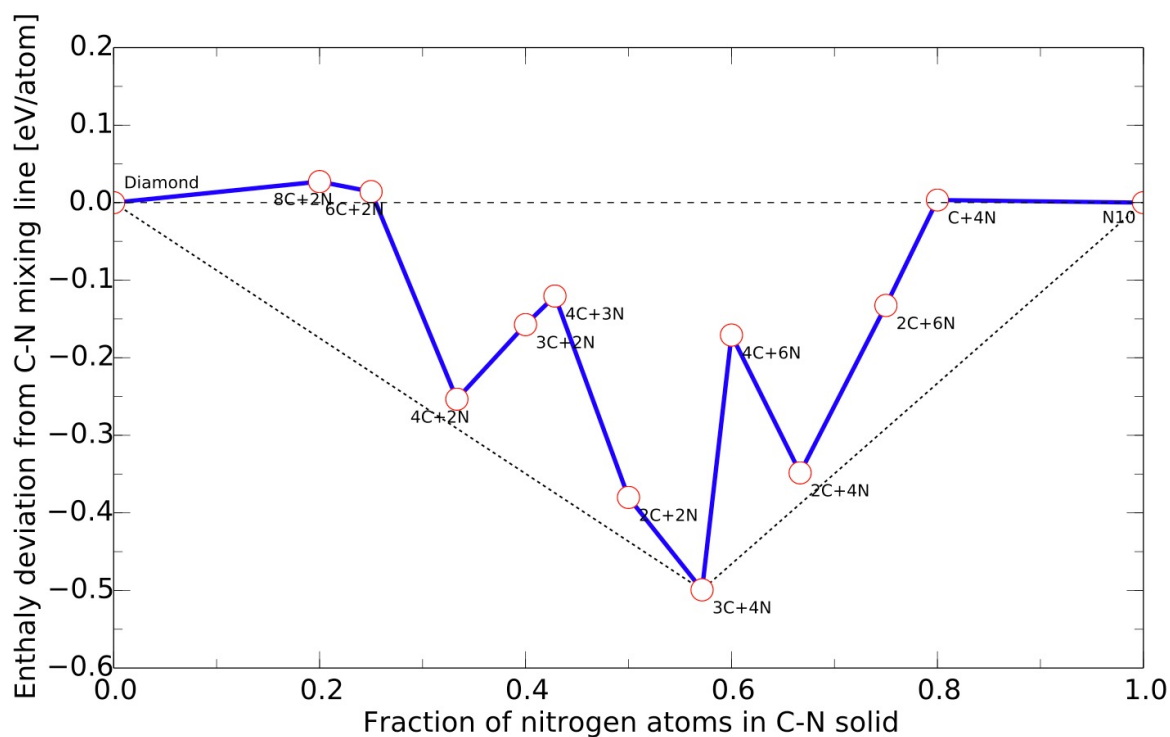
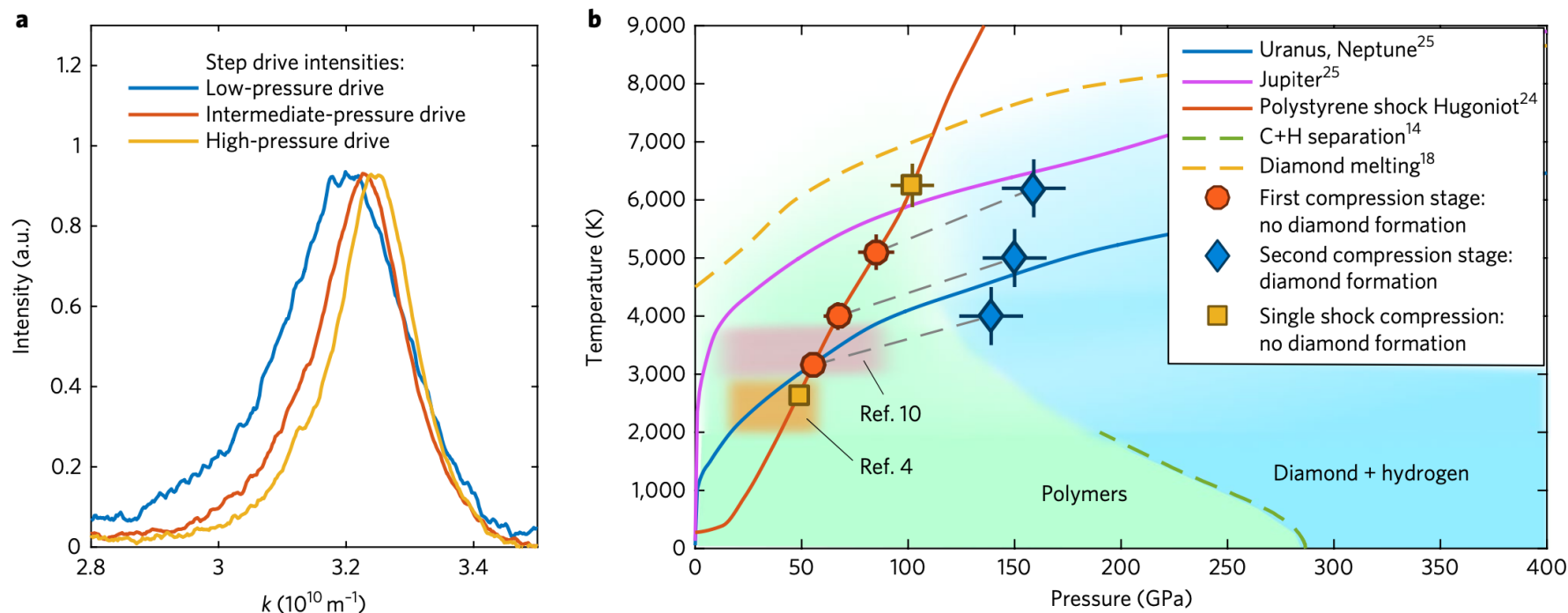


Figure 10: Convex hull diagram from our crystal structure search of C-N compounds at 3 Mbar that identified C₃N₄ as the most stable compound besides diamond and an N₁₀ structure.

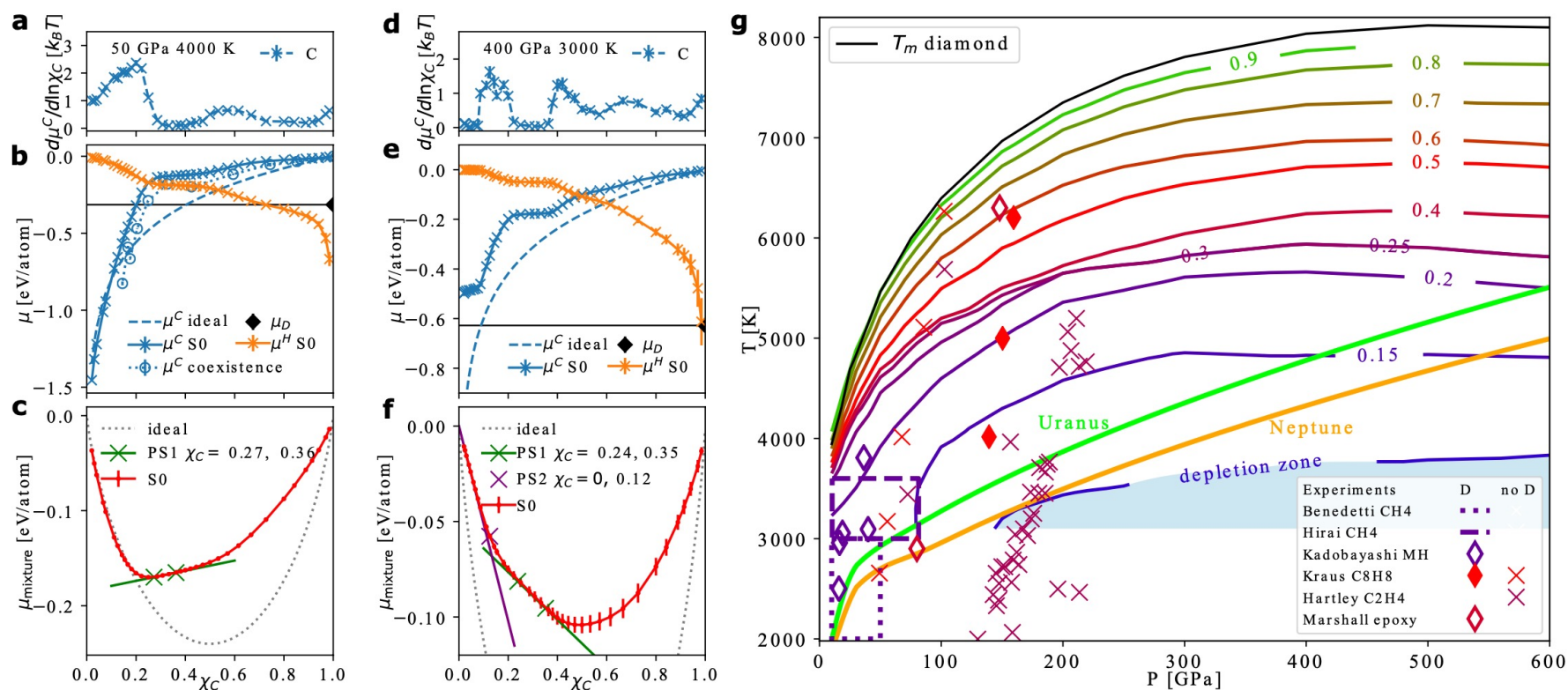
Kraus et al. (2017): Laser Compressed Hydrocarbons form Diamonds at High Pressure and Temperature

NATURE ASTRONOMY

LETTERS



Cheng, Hamel, Bethkenhagen (2022): Diamond Formation from Hydrocarbon Mixtures



Simulations Predict H_2O and H_2 are miscible. Experiments do not under some conditions.

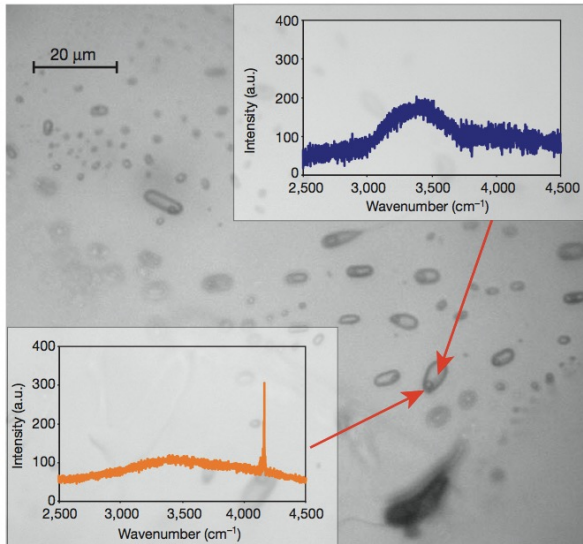
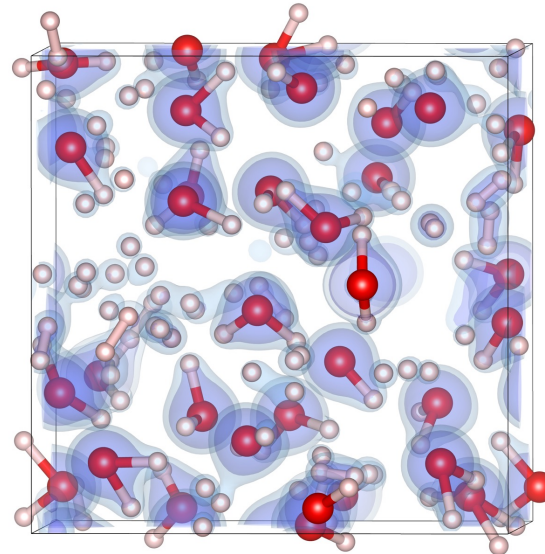
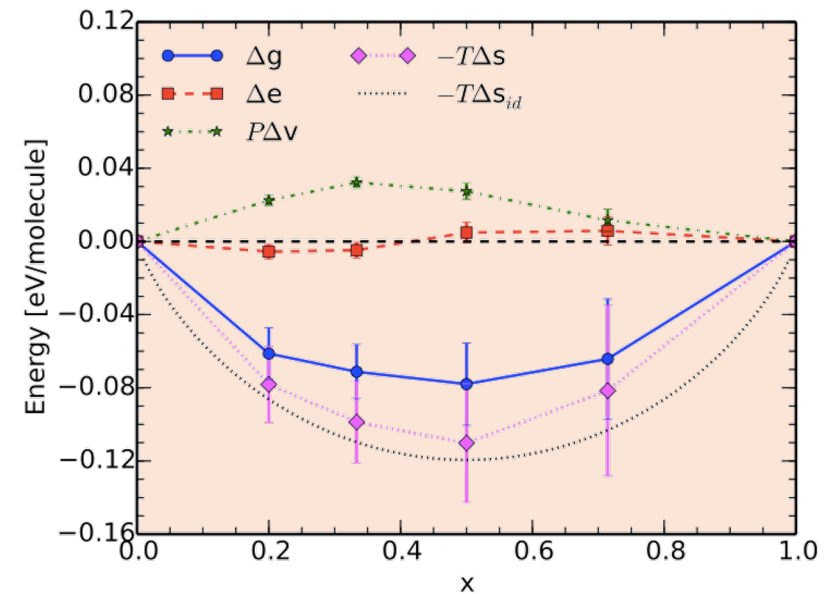


Figure 2 | Synthetic fluid inclusions in olivine formed at 950 °C and 2.3 GPa at Fe-FeO buffer conditions. Only one type of fluid inclusion is visible,

Bali, Nature (2013)



Francois Soubiran, BM, ApJ (2016)



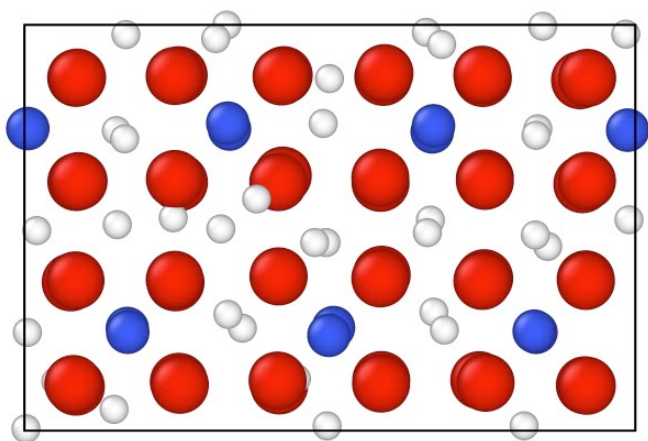
2000 K and 20 GPa

Simulations predict H_2O and H_2 to be miscible in U+N's interior.

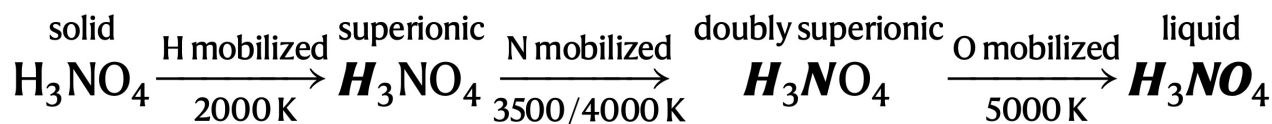
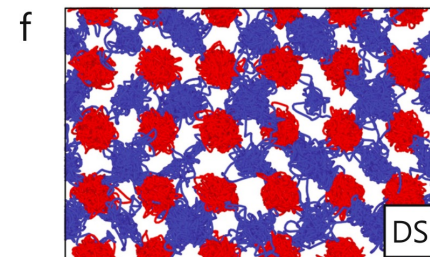
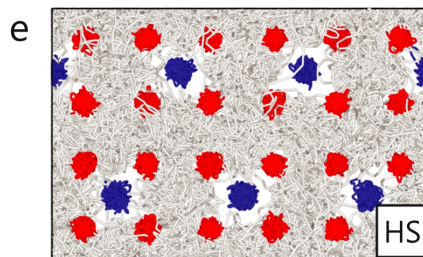
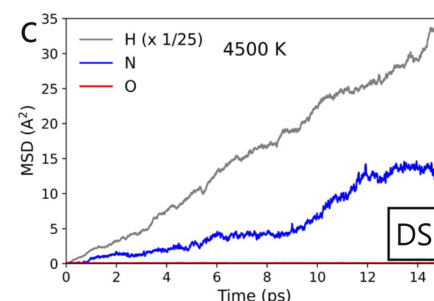
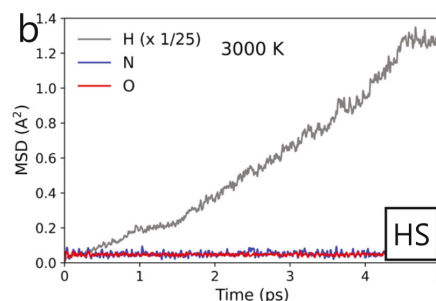
Doubly Superionic State of C-N-O-H Compounds: Hydrogen and nitrogen mobile while oxygen is not.



Kyla de Villa,
F. Gonzalez, BM,
Nat. Comm. 14
(2023) 7580

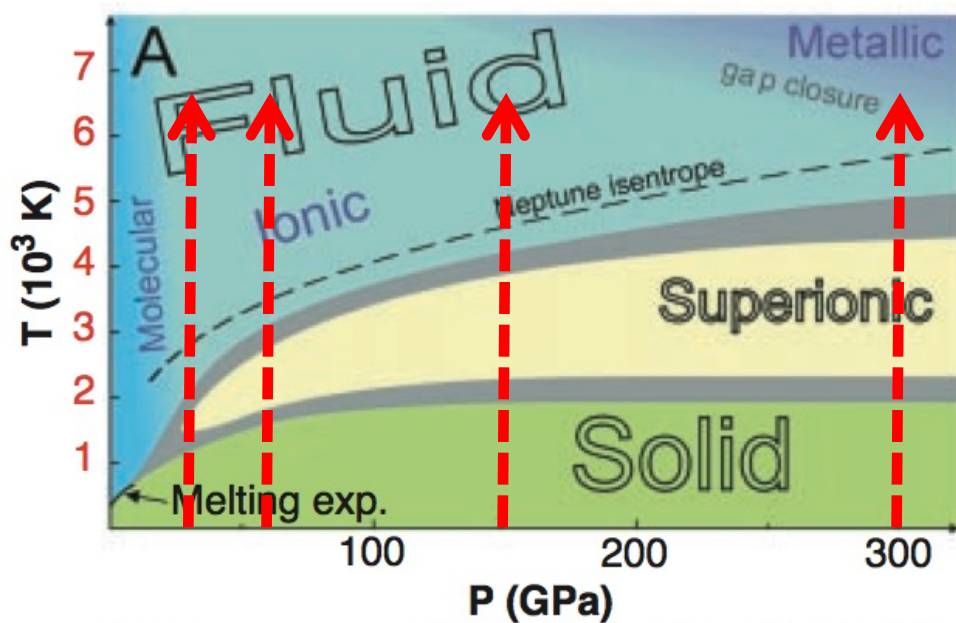


Heated up structures from Conway et al. and Naumova et al. (2021)



<http://arxiv.org/abs/2410.17499> How to detect DSI with experiments? Shock+XRD

Can Superionic Water explain the nondipolar fields?



Cavazoni et al. (1999)

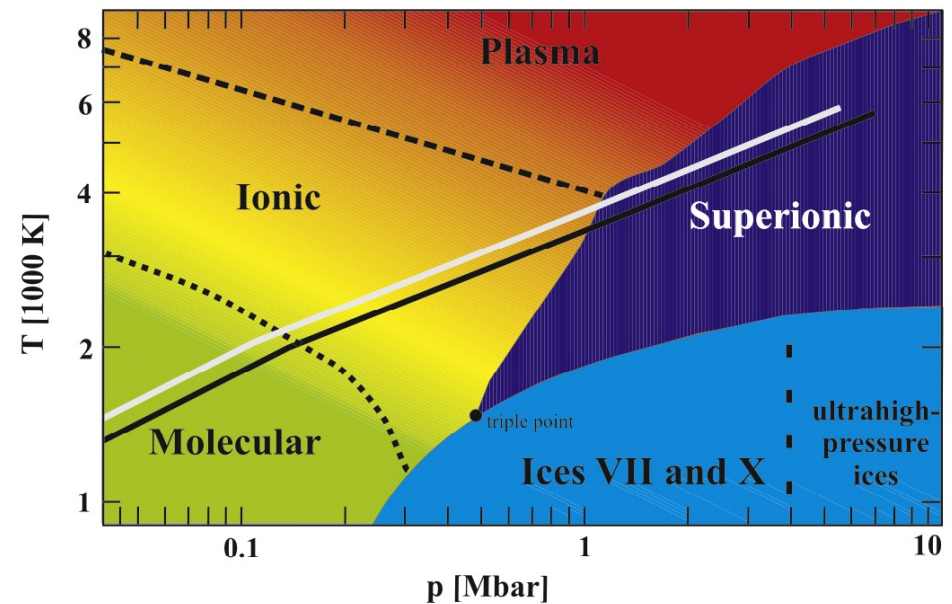
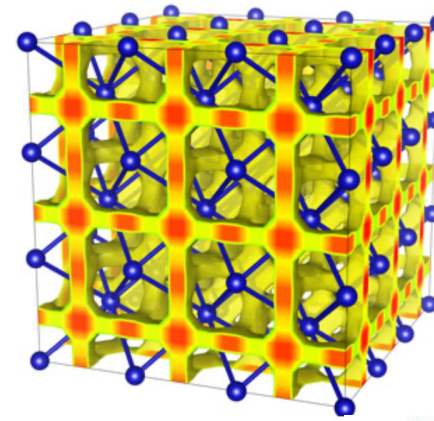
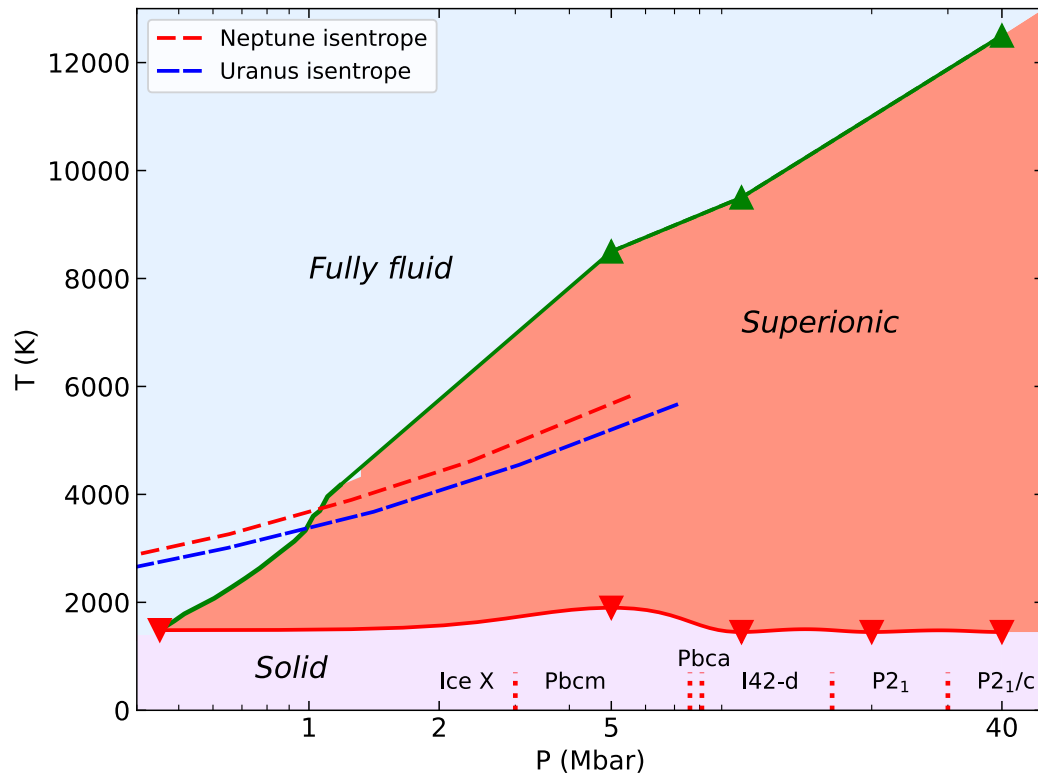


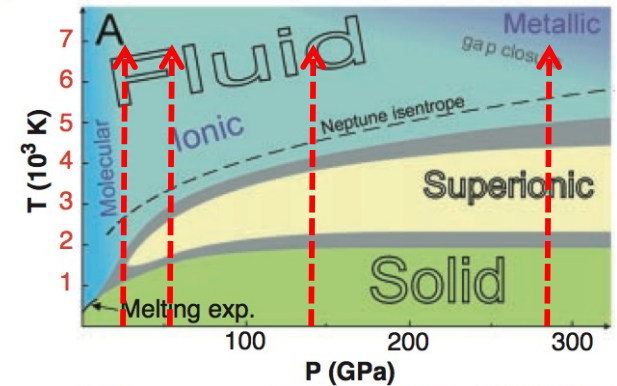
Fig. 1. Phase diagram of water up to high pressures as relevant for the interiors of Uranus and Neptune. The solid (ice VII and X), fluid (molecular, ionic, plasma), and

Redmer et al (2011)

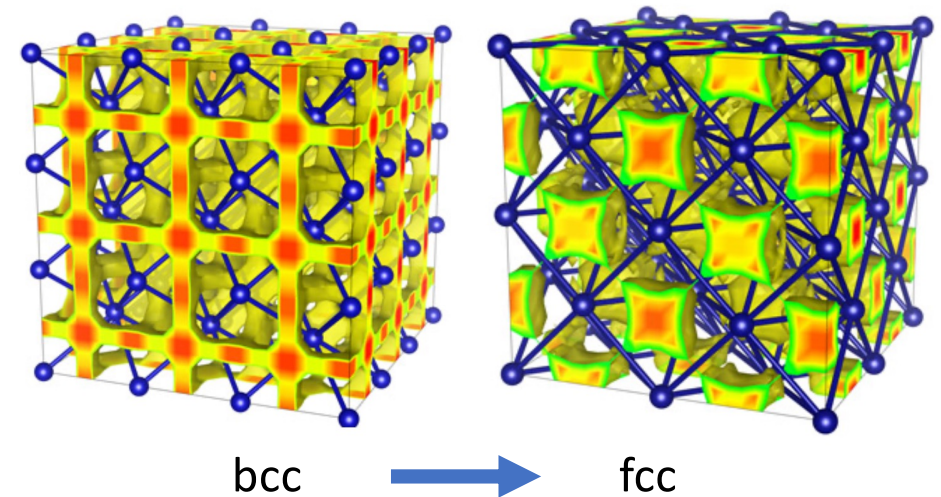
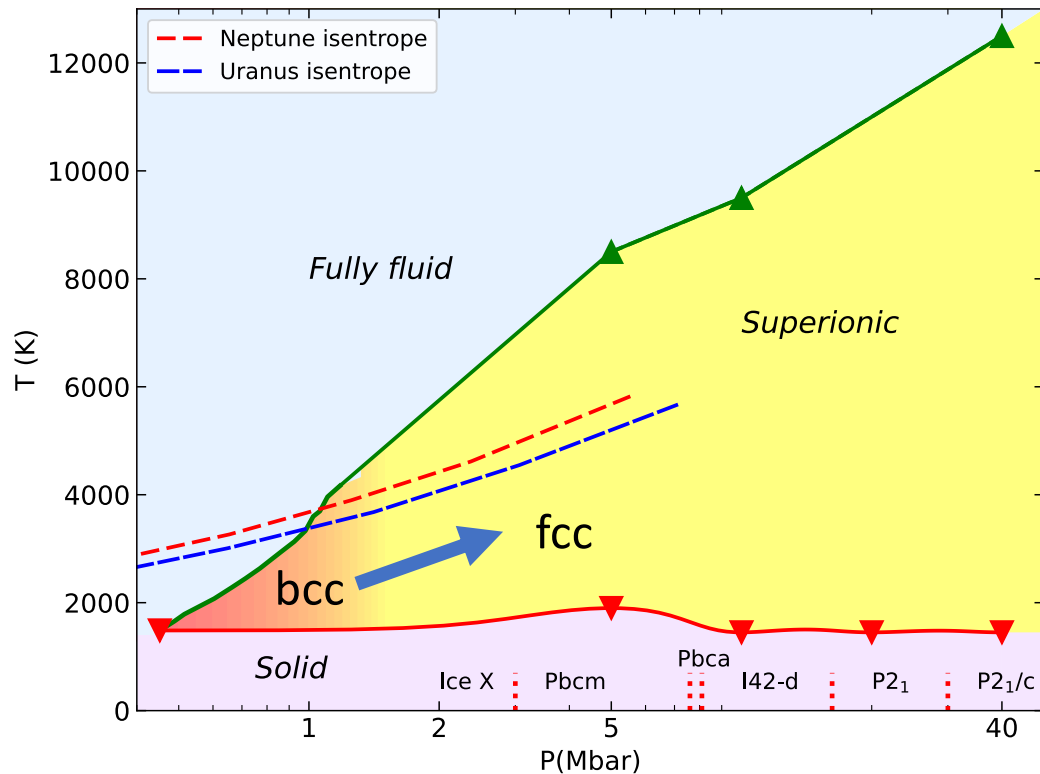
Because Cavazoni had started from ice X, all superionic calculations assumed a bcc structure



bcc



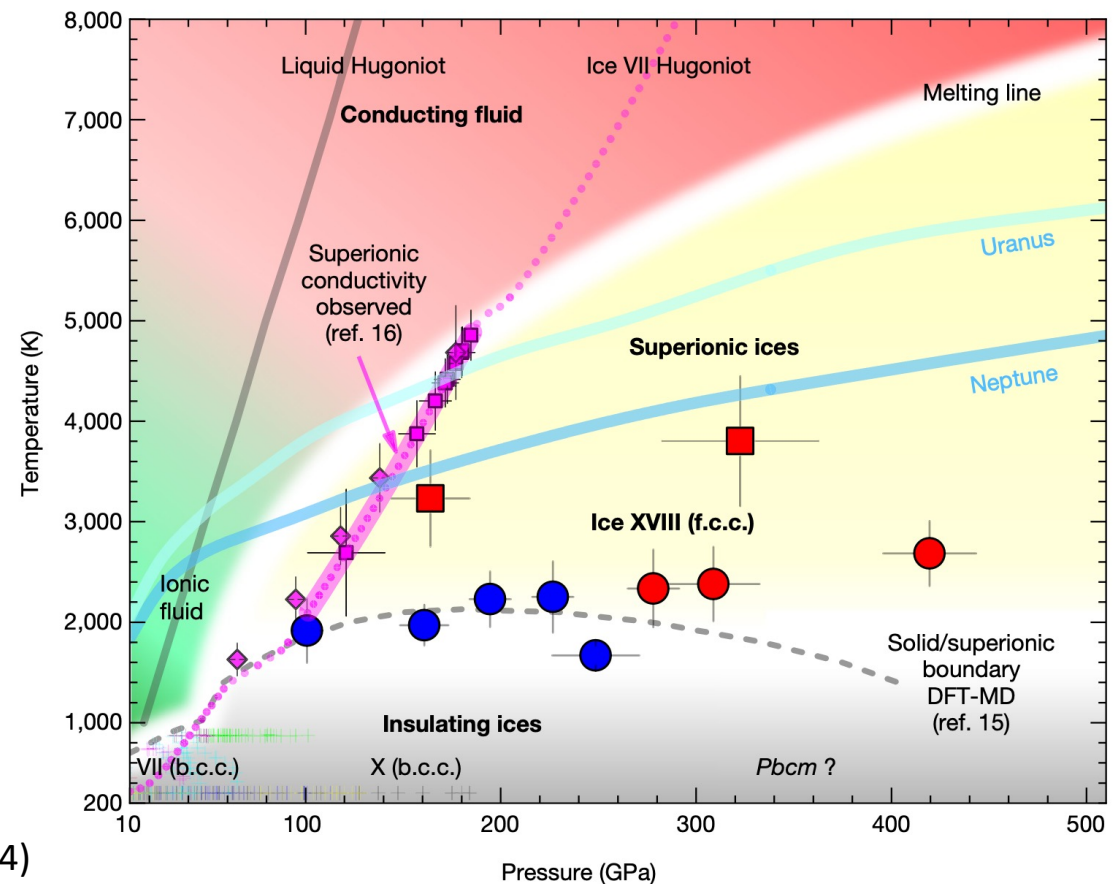
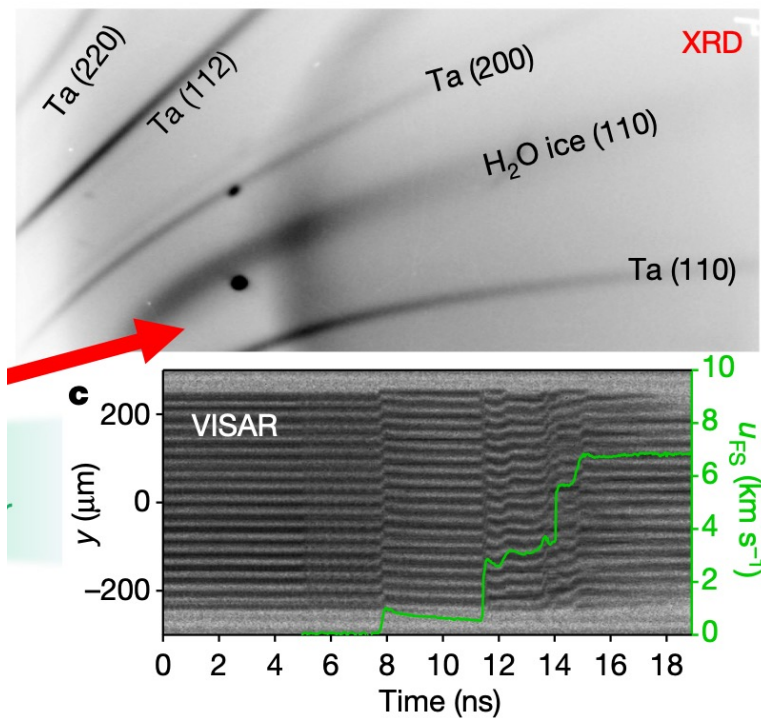
Bcc-to-fcc transition of Superionic Water Predicted with Gibbs Free Energy Calculations



Transition pressure inferred from Gibbs free energy calculations.

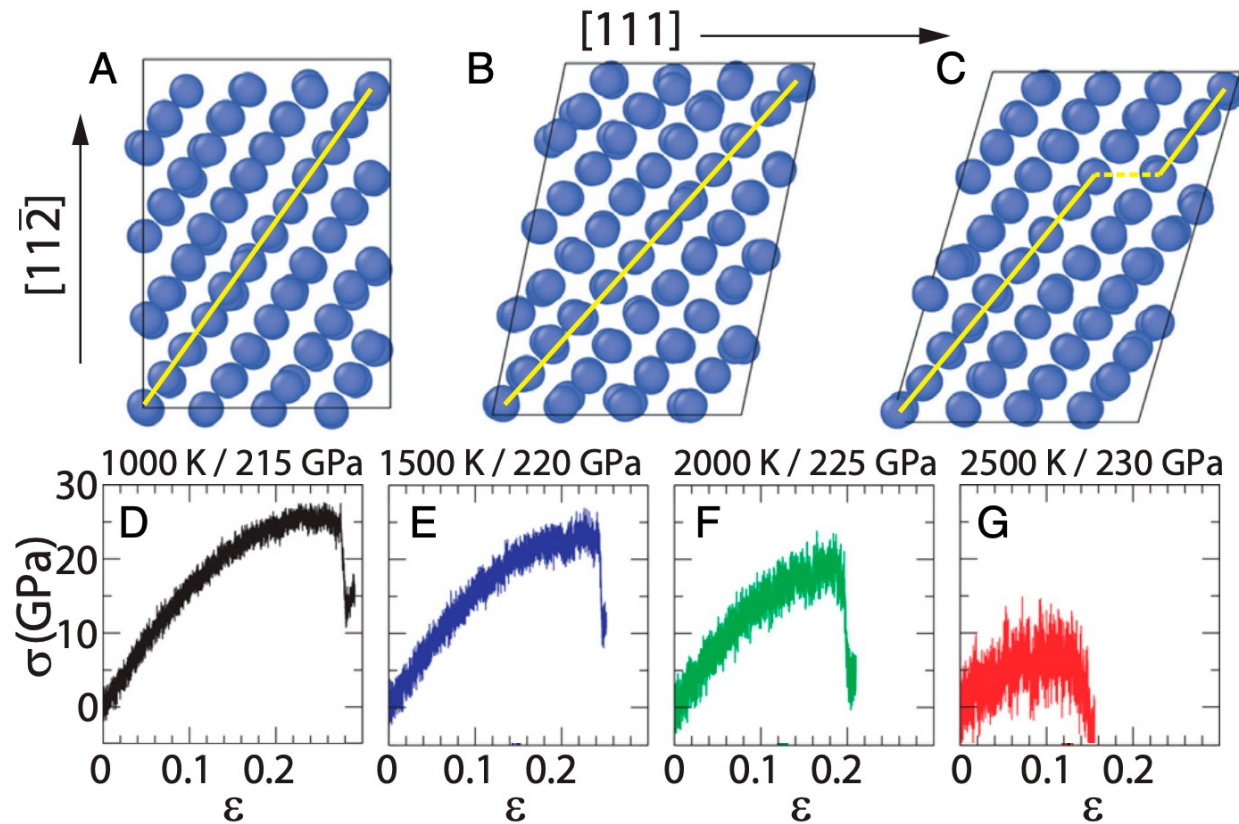
H. F. Wilson, M. L. Wong, B. Militzer, Physical Review Letters 110 (2013) 151102

Millot et al. 2019: FCC Superionic ice generated with laser shock experiments



Stay tuned for results from D. Kraus (Gordon conf, 2024)

Matsulema et al. 2022: Superionic ice flows easily



Nettelmann et al. (2016): Thermal boundary layer to explain Uranus' low luminosity

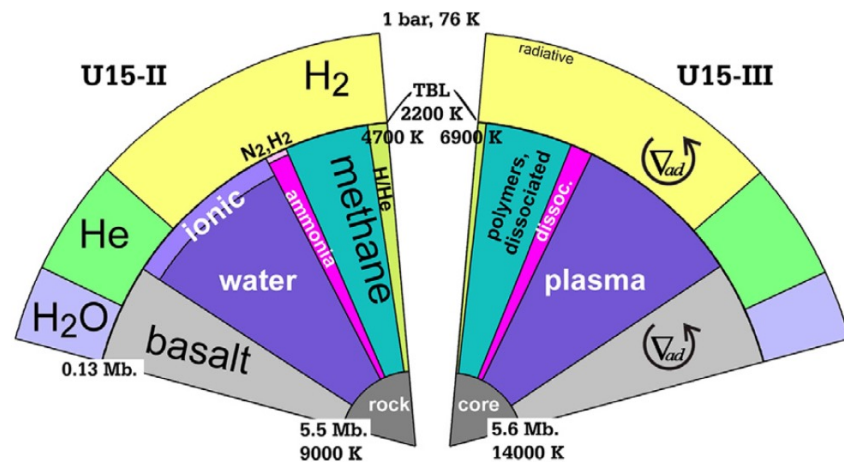
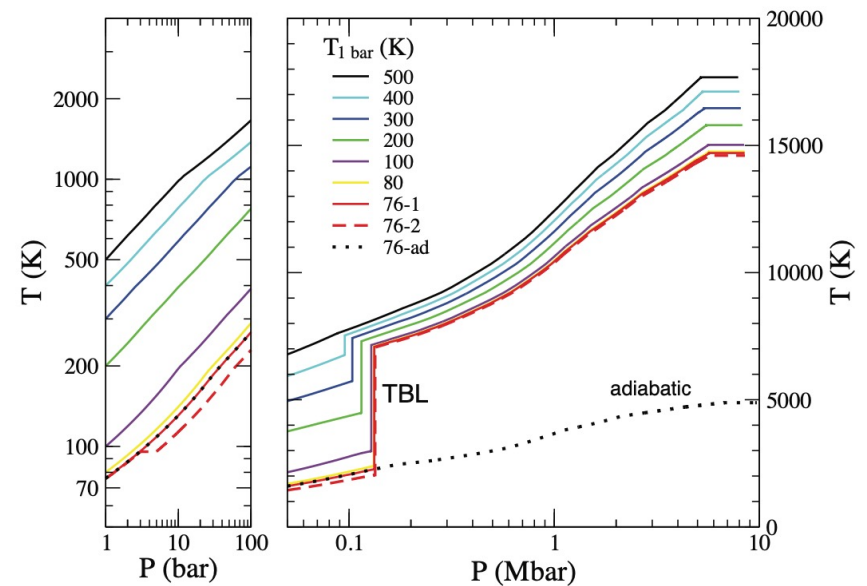


Fig. 9. Uranus three-layer structure models with thermal boundary layer that fit the gravity data and the luminosity. (Left) Model U15-II, with a maximum change of



Nonadiabatic models combine compositional gradients and much higher interior temperatures

Ice mixture H₂O: CH₄: NH₃ ≈ 7.7:4:1

Helled et al. (2020): Three layer: H, H₂O and rock; sharp and fuzzy boundaries

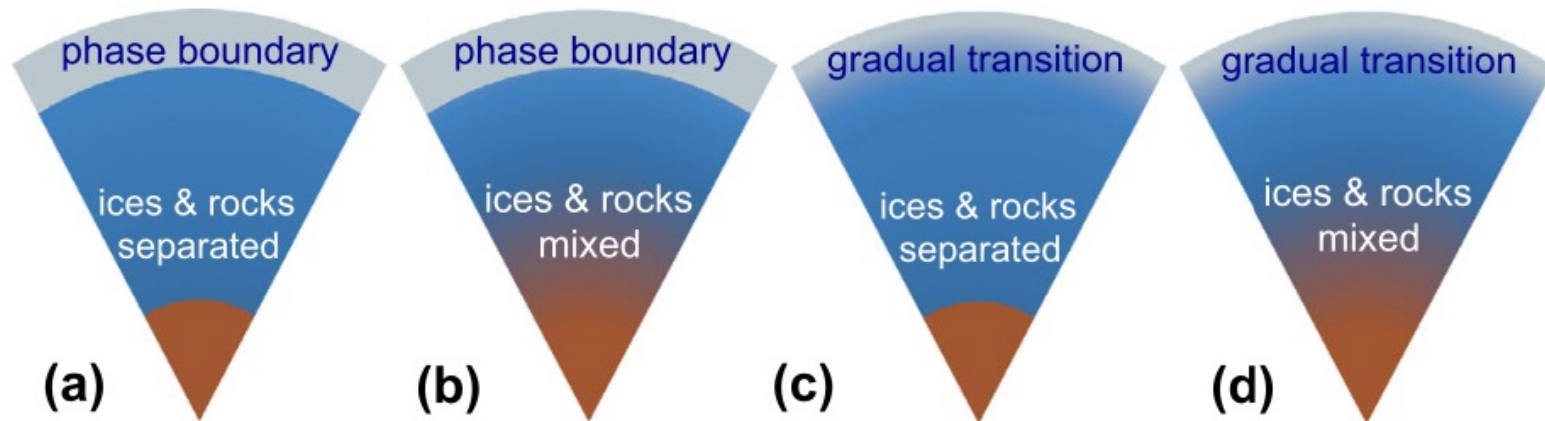
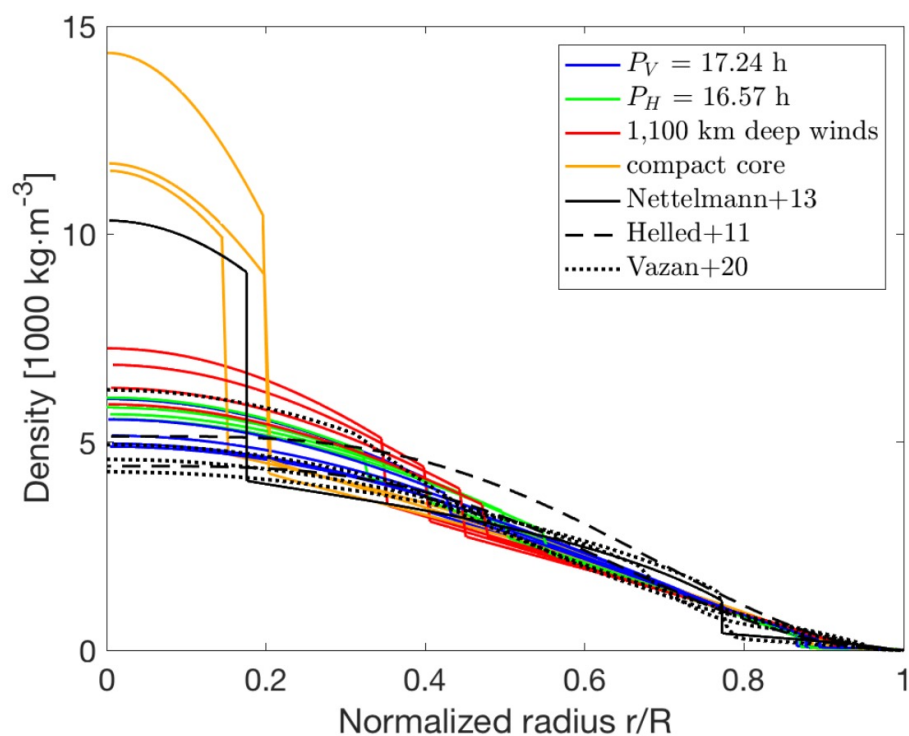
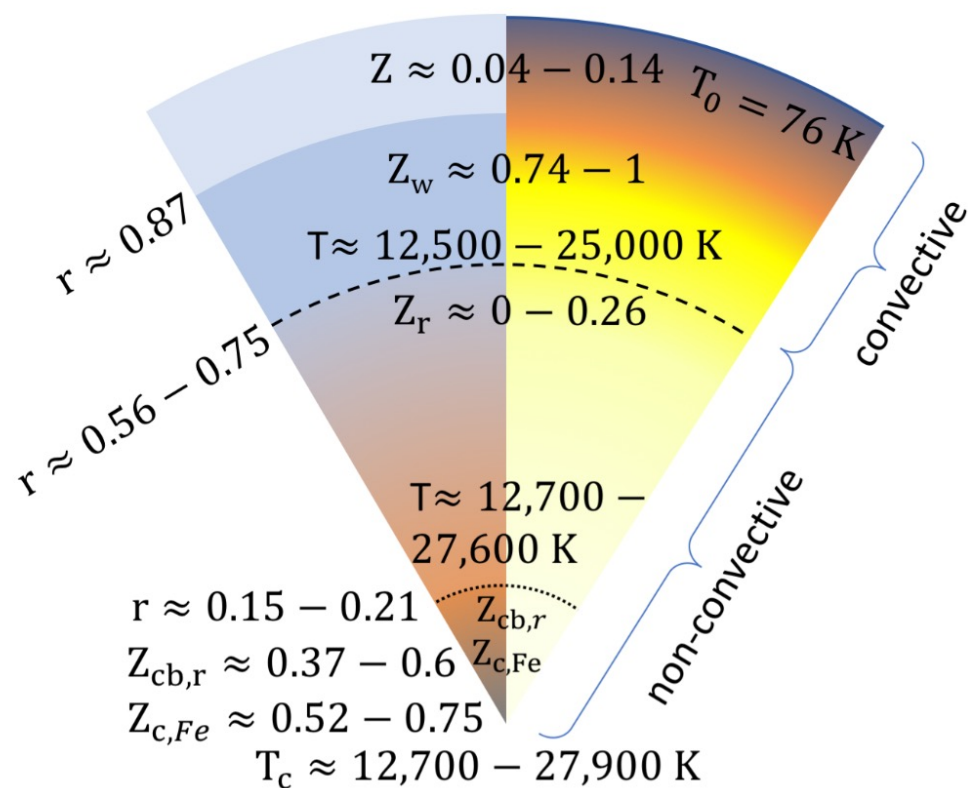


Fig. 4 Sketches of the possible internal structures of an ice giant. It is unclear whether Uranus and Neptune are differentiated and whether the transition between the different layers are distinct or gradual: (a) separation

Neuenschwander et al. (2024): Convective/Non-convective models with high interior temperatures



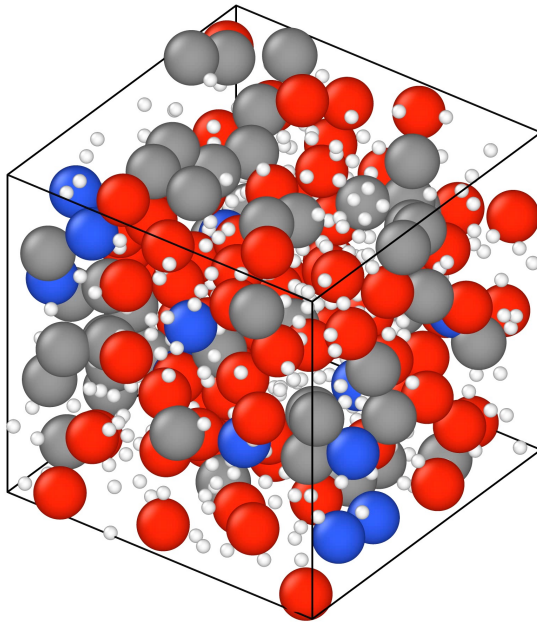
H-He, water, rock & iron, $P_V = 17.24 \text{ h}$





Machine learning becomes popular. These methods learn forces from ab initio simulations and then makes them much faster

Performed much bigger simulations with Machine-Learning Accelerated Ab initio Simulations: 540 atoms

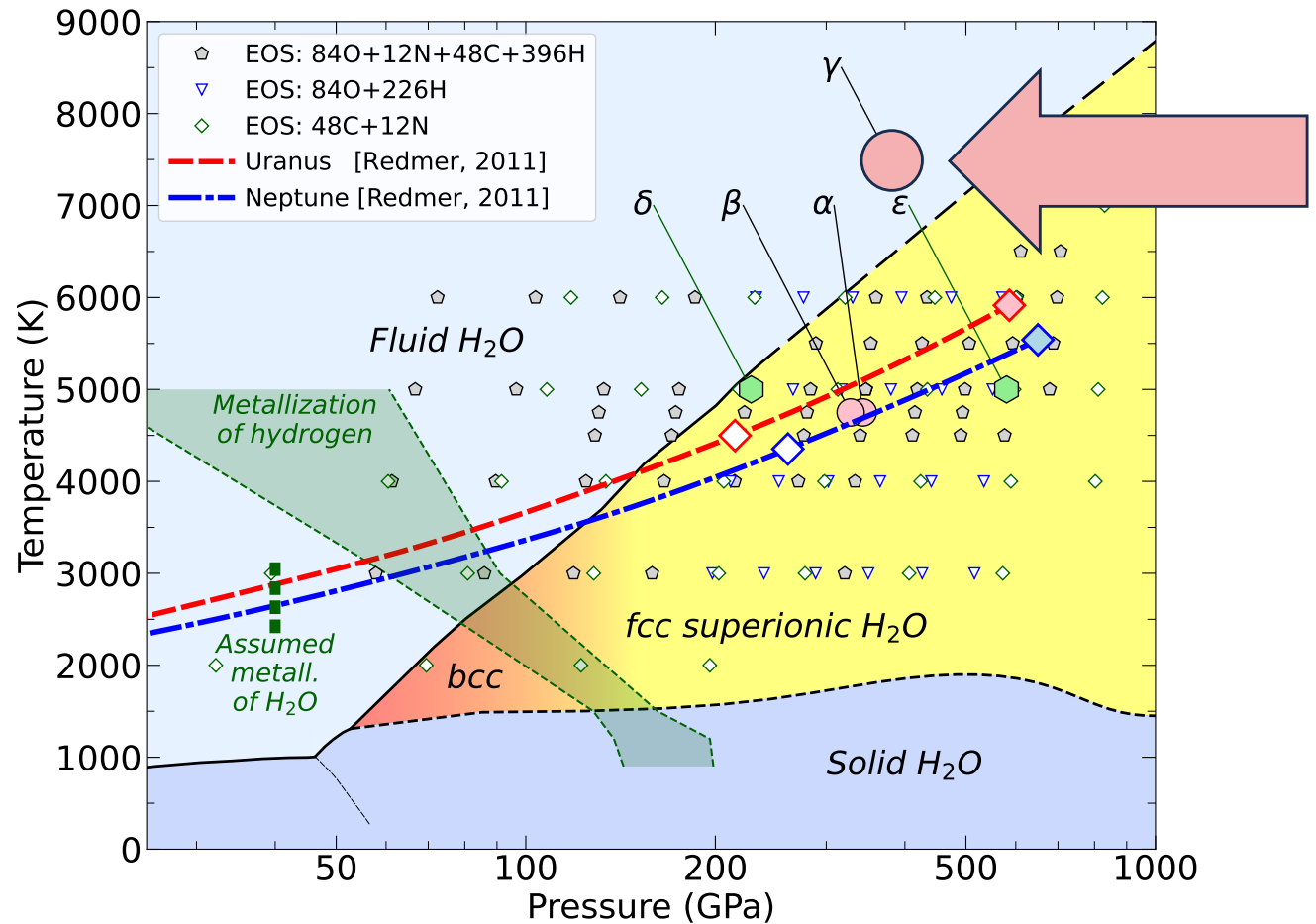


540 atoms:

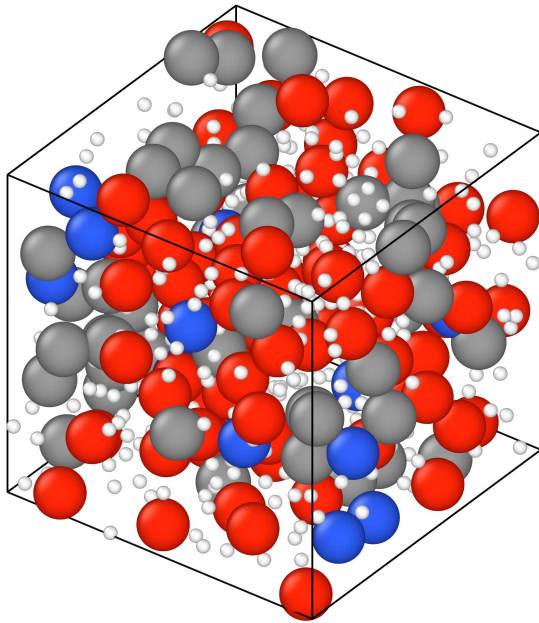
$84\text{O} + 48\text{C} + 12\text{N} + 396\text{H} =$

$12 \times [7\text{H}_2\text{O} + 4\text{CH}_4 + \text{NH}_3]$

$\text{O}:\text{C}:\text{N} = 7:4:1$



Performed much bigger simulations with Machine-Learning Accelerated Ab initio Simulations: 540 atoms

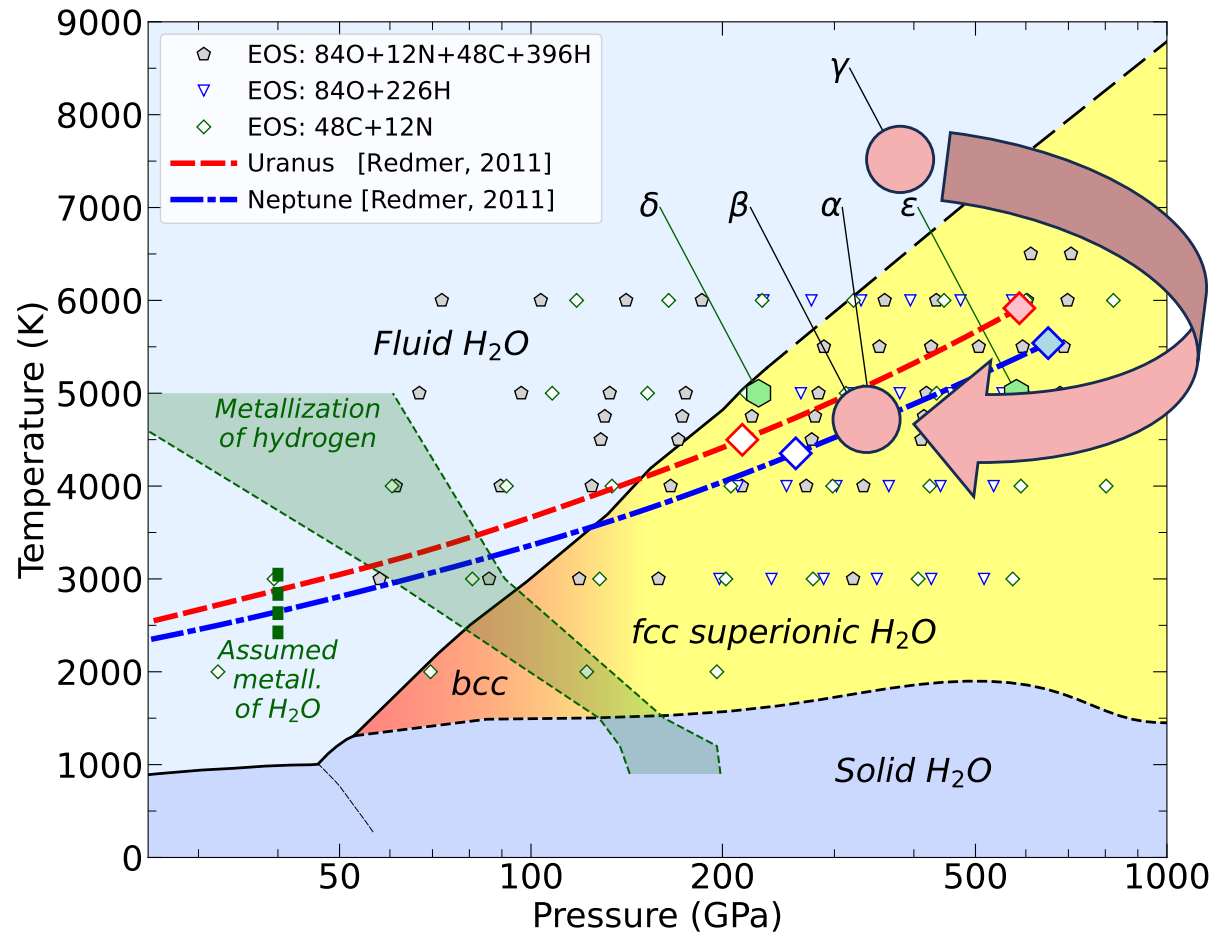


540 atoms:

$84\text{O} + 12\text{N} + 48\text{C} + 396\text{H} =$

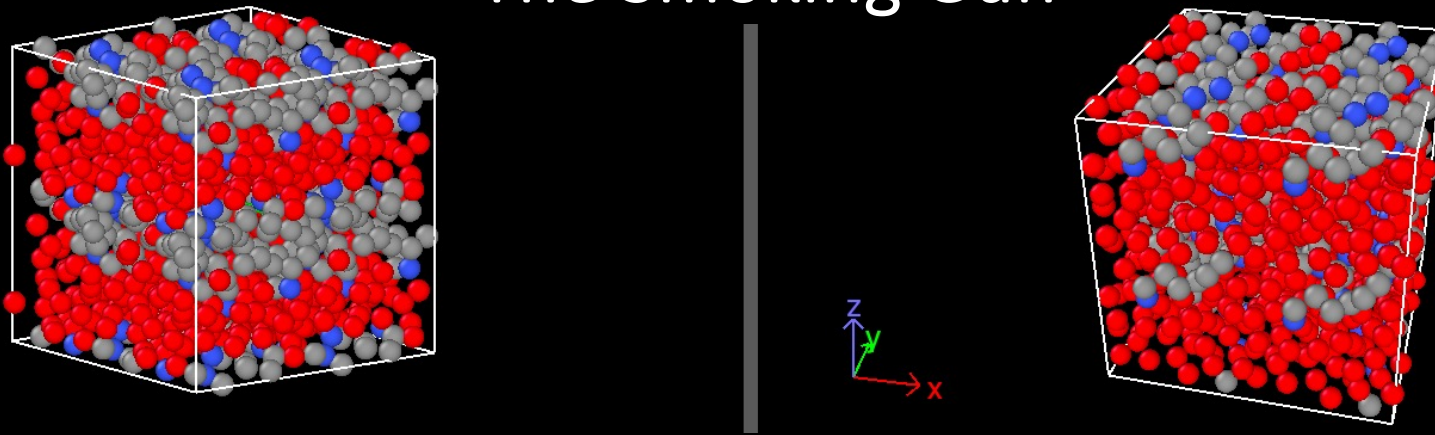
$12 \times [7\text{H}_2\text{O} + 4\text{CH}_4 + \text{NH}_3]$

$\text{O}:\text{C}:\text{N} = 7:4:1$



Smoking gun came from simulation with $84\text{O}+12\text{N}+48\text{C}+396\text{H} = 12 \times [7\text{H}_2\text{O} + 4\text{CH}_4 + \text{NH}_3]$

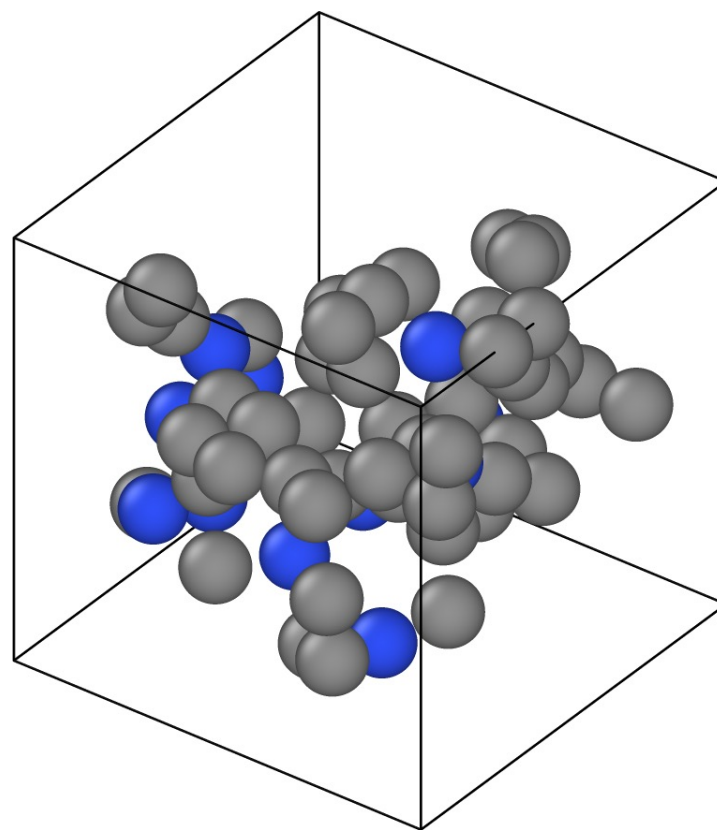
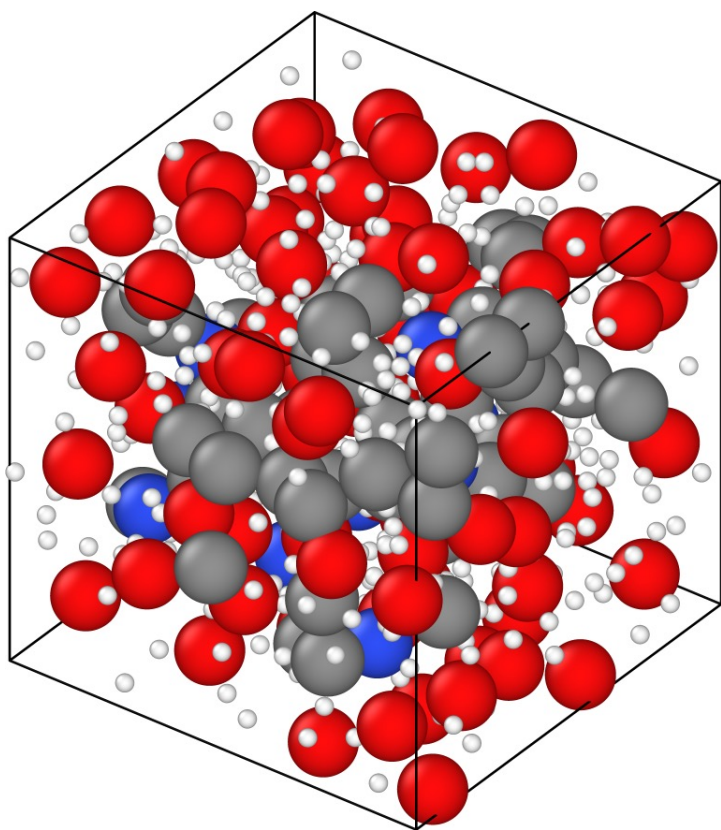
The Smoking Gun



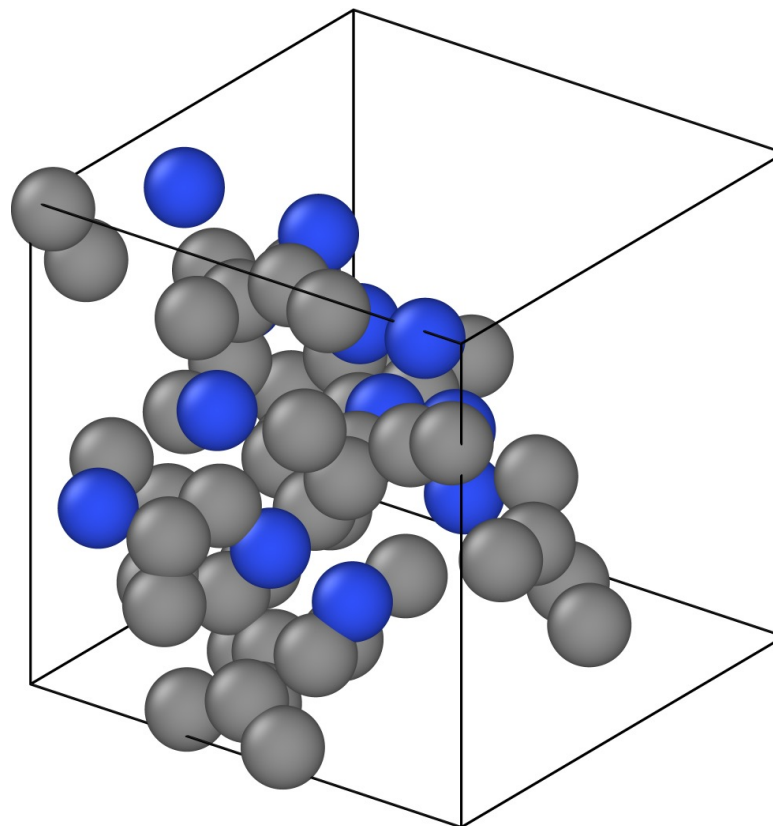
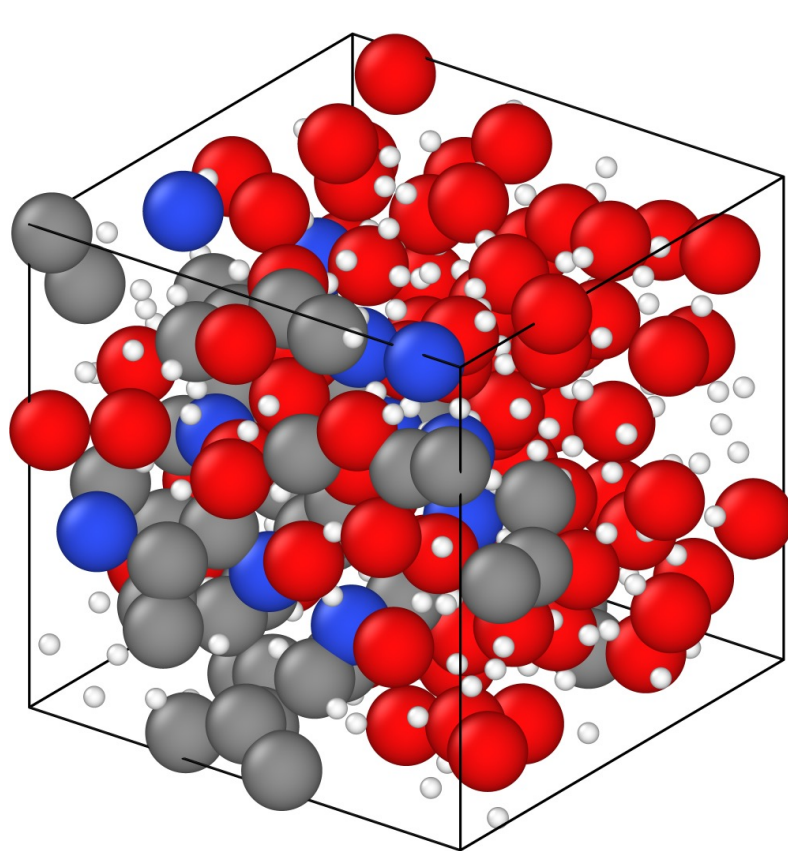
Is this real?

- 1) Would anyone confirm this result with ab initio simulations? So we turned off all machine learning and reconfirmed the findings!
- 2) Can this be confirmed with laboratory experiments?
- 3) Will a Uranus orbiter detect a signature in the planet?

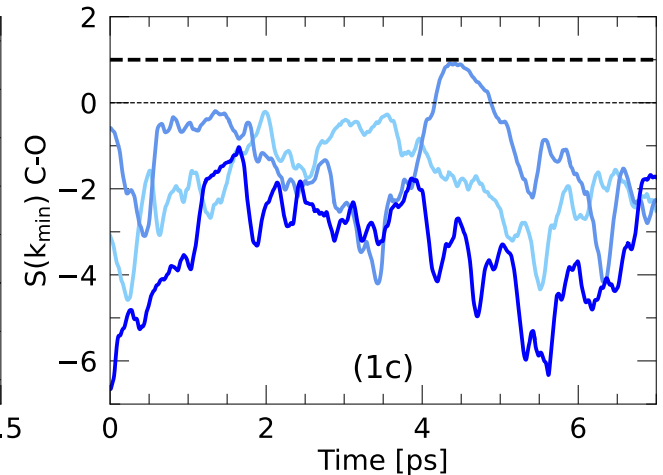
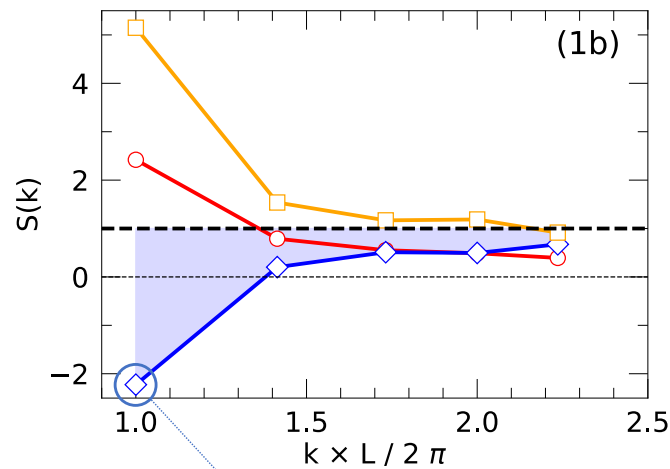
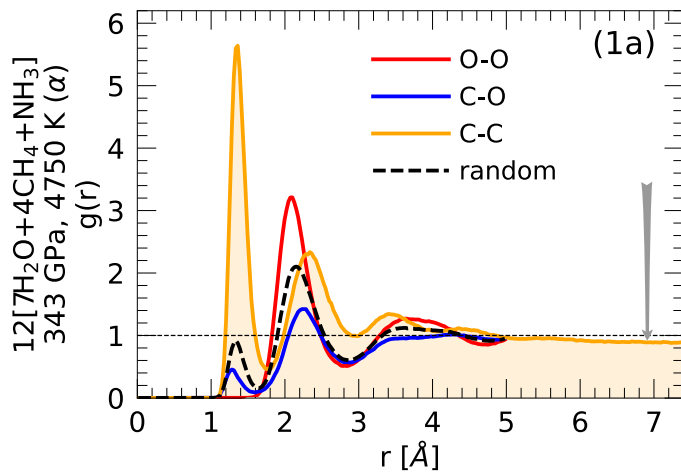
Phase separation in simulation with
 $84\text{O} + 12\text{N} + 48\text{C} + 396\text{H} = 12 \times [7\text{H}_2\text{O} + 4\text{CH}_4 + \text{NH}_3]$



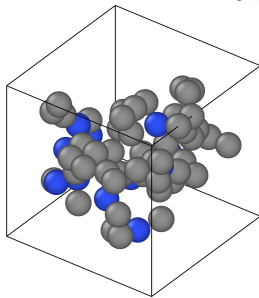
Phase Separation Confirmed with **H-depleted** simulations
84O+12N+48C+232H (164 H atoms were removed)



Pair Correlation $g(r)$ and Structure $S(k)$ confirm phase separation

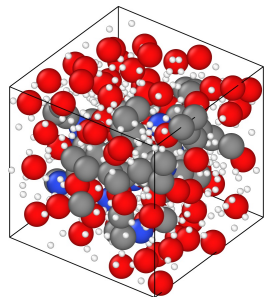
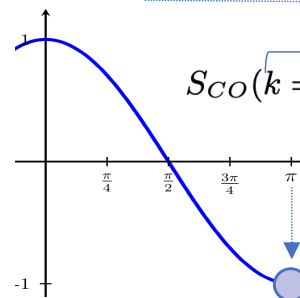


$$g_{AB}(r) = \frac{V}{4\pi r^2 N_A N_B} \sum_{i=1}^{N_A} \sum_{j=1}^{N_B} \langle \delta(|\vec{r}_i - \vec{r}_j| - r) \rangle$$

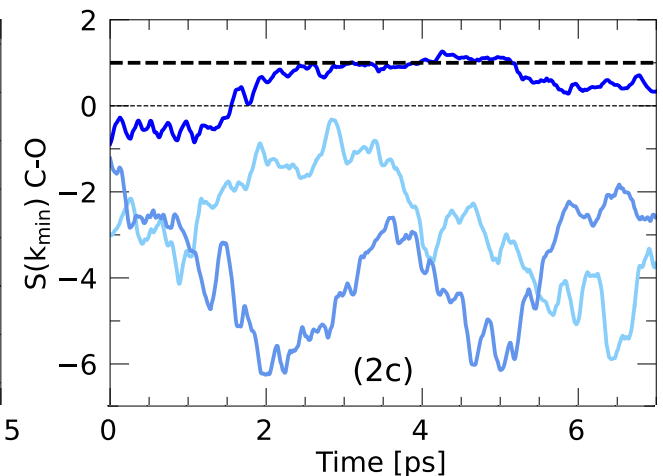
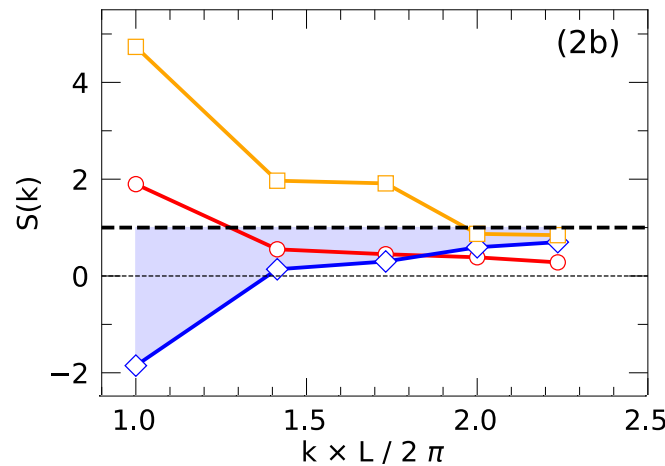
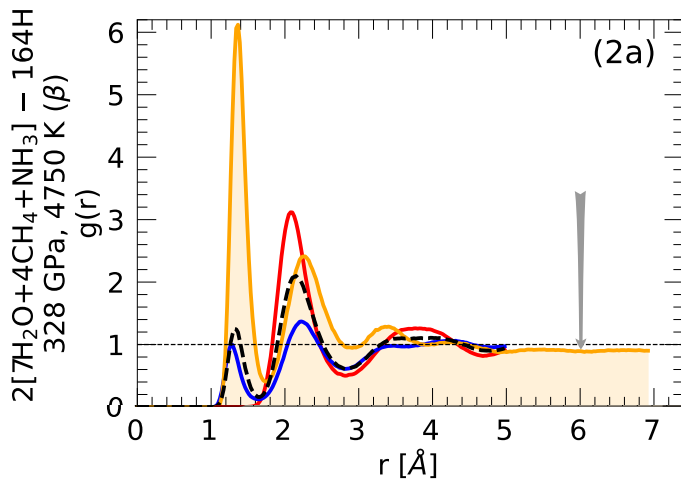
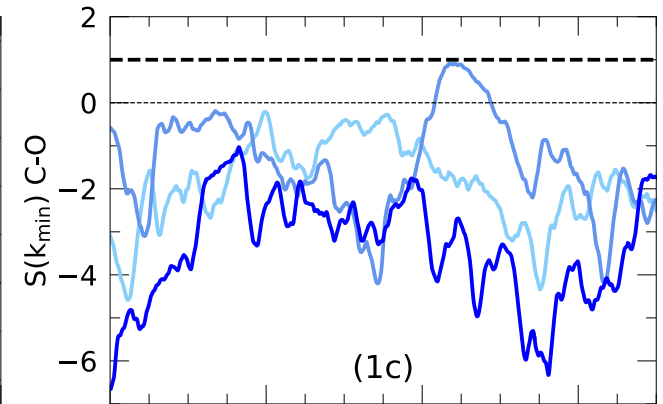
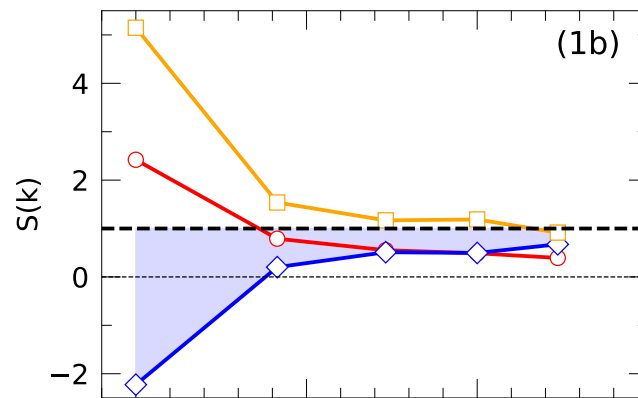
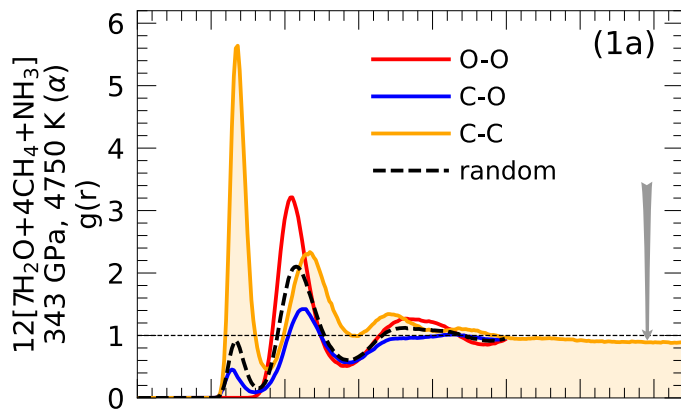


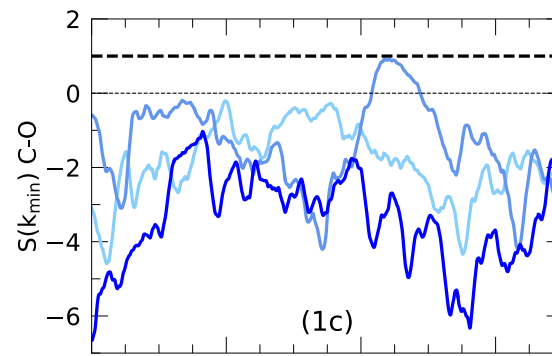
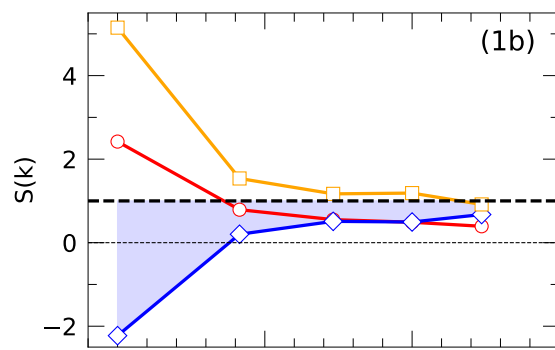
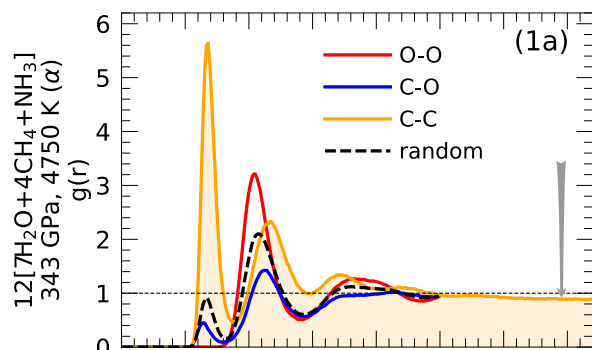
$$S_{AB}(\vec{k}) = \frac{1}{\sqrt{N_A N_B}} \sum_{i=1}^{N_A} \sum_{j=1}^{N_B} \langle \exp \{ -i\vec{k}(\vec{r}_i - \vec{r}_j) \} \rangle$$

$$S_{CO}(k = 2\pi/L) \sim \sum_C^{N_C} \sum_O^{N_O} \cos \left(2\pi \frac{x_C - x_O}{L} \right)$$

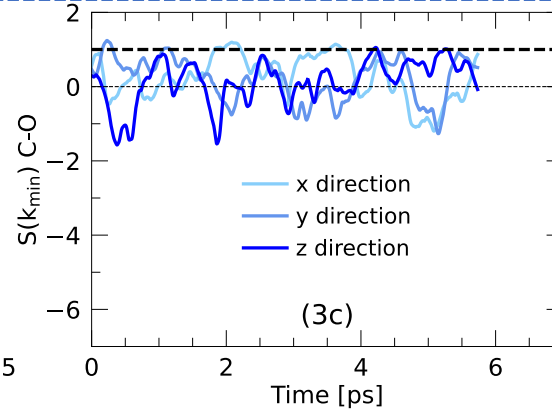
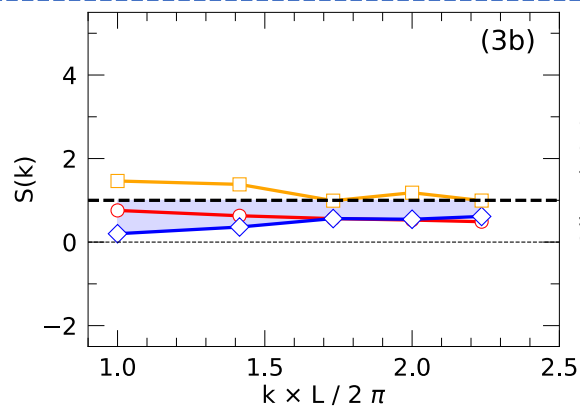
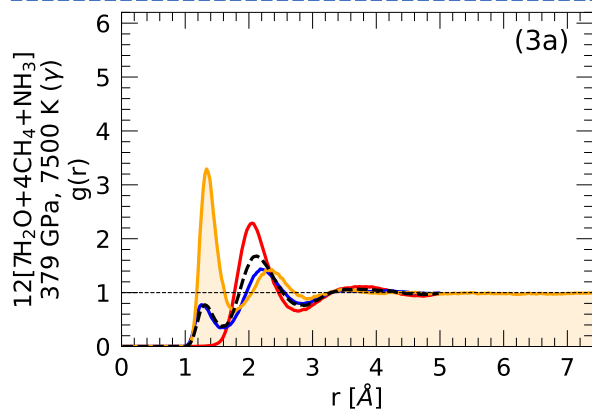
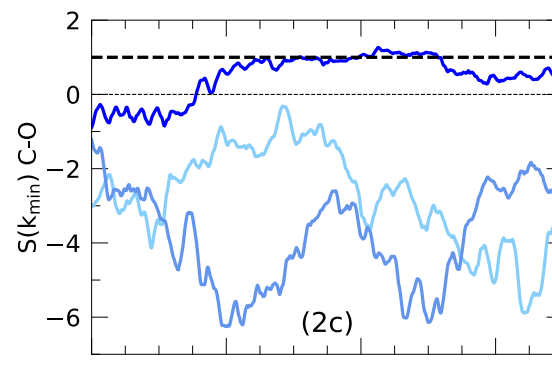
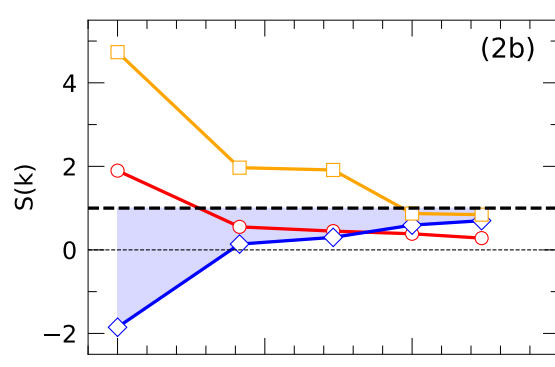
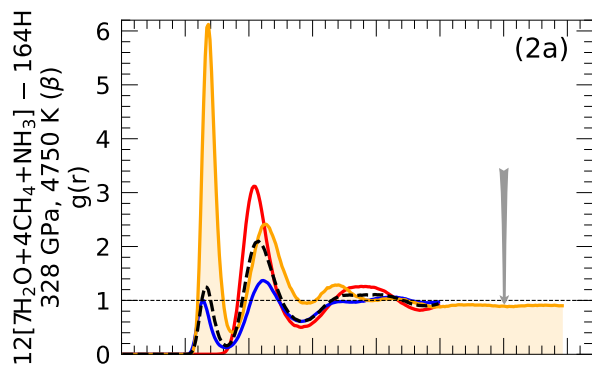


Pair Correlation $g(r)$ and Structure $S(k)$ confirm phase separation





At 4750 K,
one sees
evidence of
phase
separation



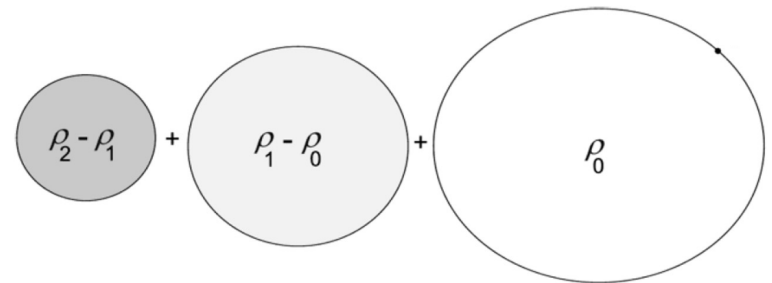
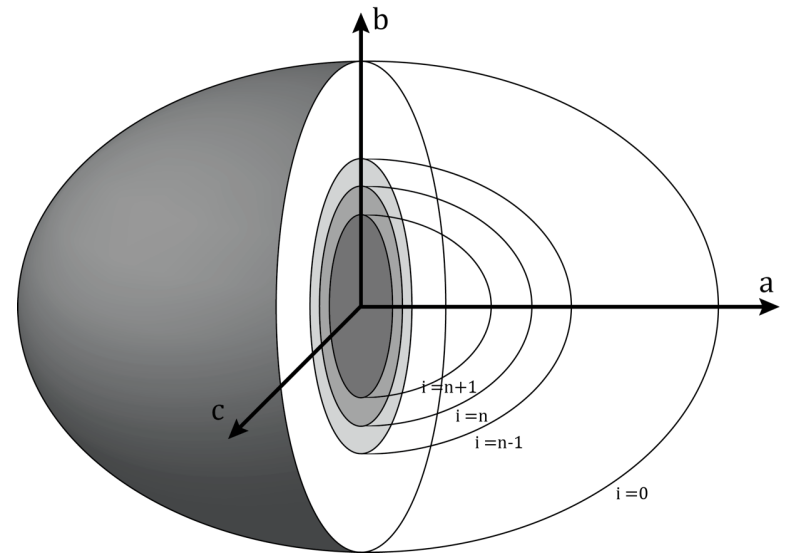
At 7500 K, the
fluid remains
well mixed.

Concentric Maclaurin Spheroid (CMS) theory for rotating bodies in hydrostatic equilibrium

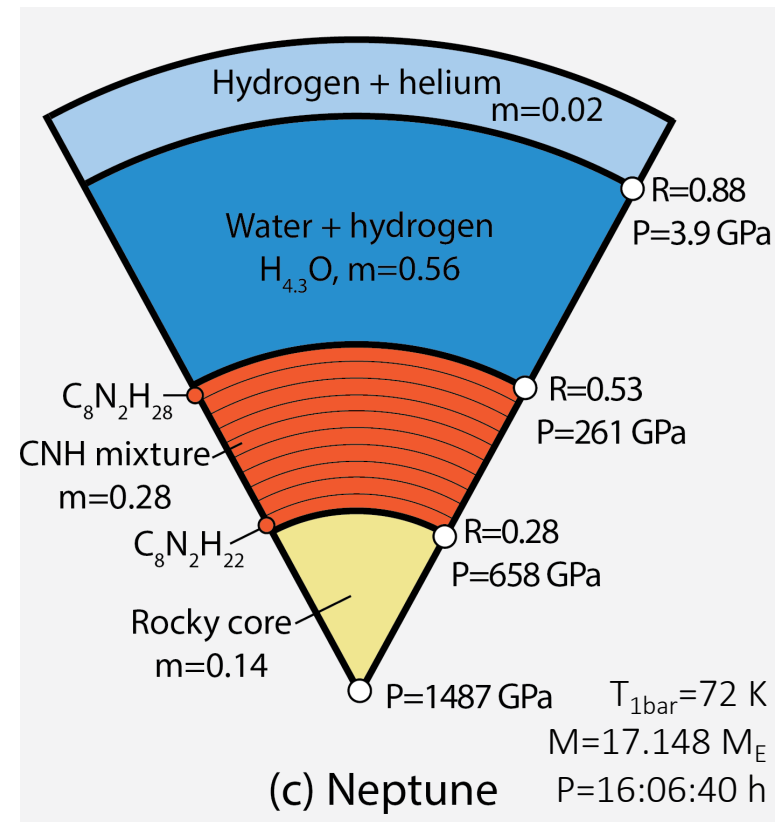
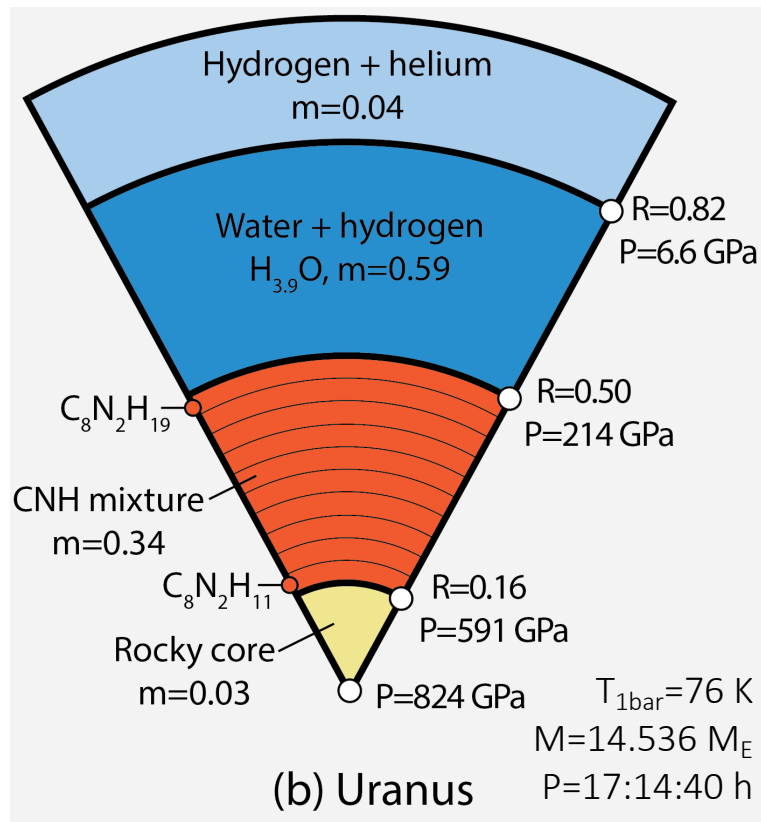
Model parameters:

- EOS from *ab initio* simulations:
 $\rho = \rho(P, T, \text{composition})$
- Locations of the **boundaries** between the four layers
- Hydrogen fraction in the **H₂O+H layer**
- Hydrogen fraction in the **C-N-H layer**

- Use CMS to compute R, M, J_2, J_4, J_6 with high accuracy.
- Concentric Maclaurin Spheroid method
Hubbard, ApJ (2013)
- Accelerated CMS method,
Militzer et al., ApJ (2019)



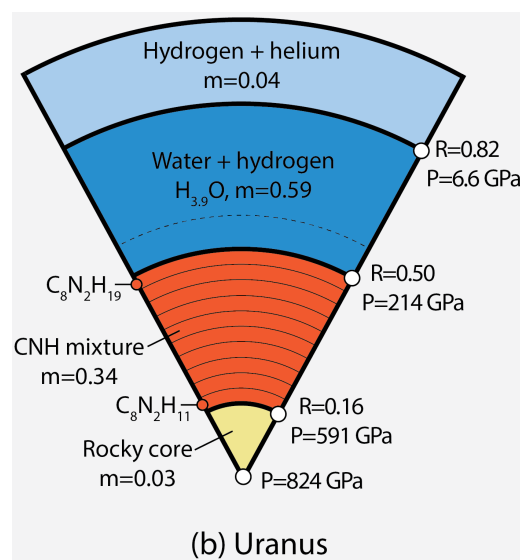
Currently Favored Interior Models



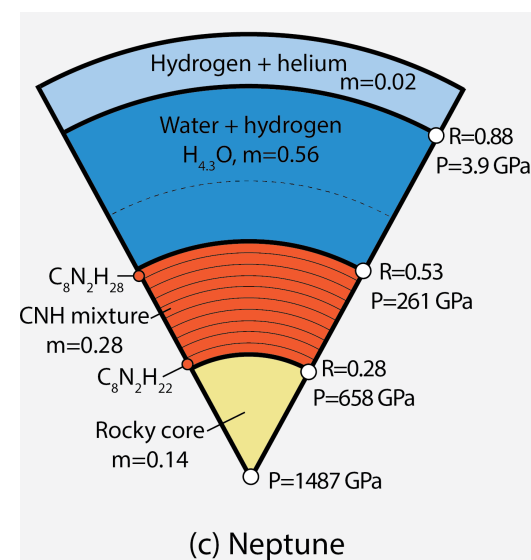
Assumptions for Interior Structure Models

Simplifying model assumptions

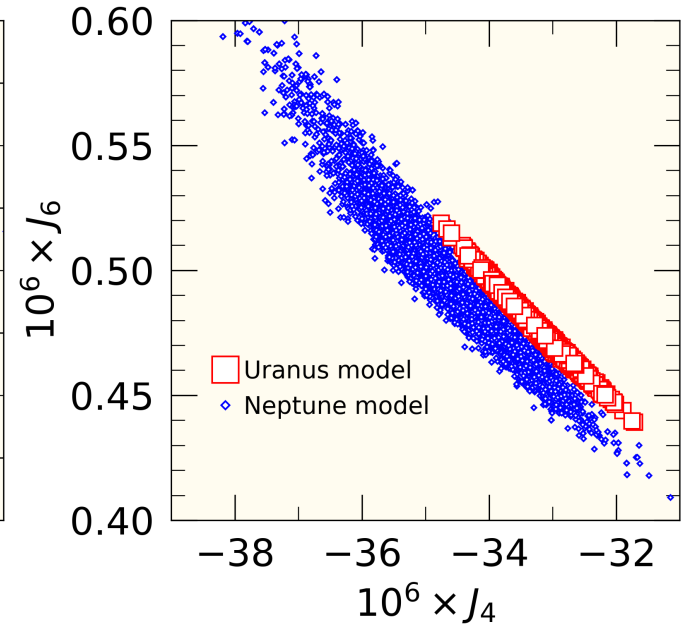
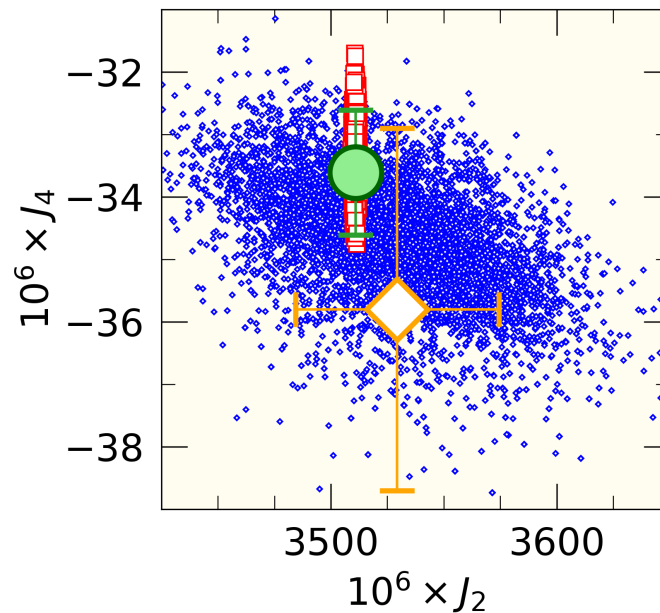
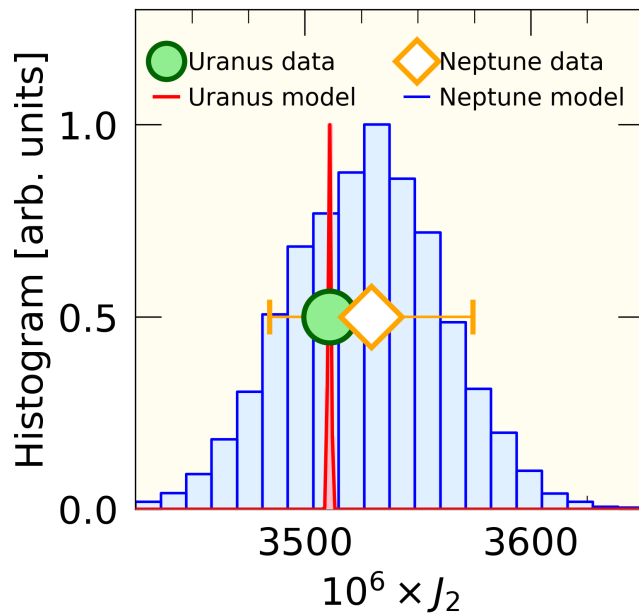
- 1) **Complete phase separation** between **H₂O-rich layer** and **C-N-H layer**
- 2) **H₂O and H₂ mix** and form a homogeneous and convecting layer
- 3) H₂O-rich layer is on top of **C-N-H layer** (So oil does not always float on top of water.)
- 4) H₂O-rich layer received its **extra hydrogen** from C-N-H but may not have absorbed it all.



| | Uranus | Neptune |
|--|--|--|
| Measured $J_2 \times 10^6$ | 3510.99 ± 0.72 | 3529 ± 45 |
| Model $J_2 \times 10^6$ | 3510.99 | 3529.40 |
| Measured $J_4 \times 10^6$ | -33.61 ± 1 | -35.8 ± 2.9 |
| Model $J_4 \times 10^6$ | -33.61 | -35.80 |
| Model $J_6 \times 10^6$ | 0.4859 | 0.5314 |
| H_1 | $1.923 \approx \text{H}_{3.8}\text{O}$ | $2.245 \approx \text{H}_{4.5}\text{O}$ |
| H_2 | $0.5015 \approx \text{C}_8\text{N}_2\text{H}_{19}$ | $0.4418 \approx \text{C}_8\text{N}_2\text{H}_{17}$ |
| H_3 | $0.2053 \approx \text{C}_8\text{N}_2\text{H}_8$ | $0.1055 \approx \text{C}_8\text{N}_2\text{H}_4$ |
| r_1 [PU] | 0.8156 | 0.8858 |
| r_2 [PU] | 0.4897 | 0.5232 |
| r_3 [PU] | 0.1471 | 0.2159 |
| r_2/r_1 (volumetric radii) | 0.6010 | 0.5915 |
| $r_2/r_{40 \text{ GPa}}$ (volumetric radii) | 0.6680 | 0.6613 |
| $\left(\frac{M_C + M_N}{M_O + M_C + M_N}\right)$ | 0.373 | 0.400 |
| $M_{\text{H absorbed}}$ [PU] | 0.05606 | 0.06902 |
| $M_{\text{H released}}$ [PU] | 0.05607 | 0.06906 |



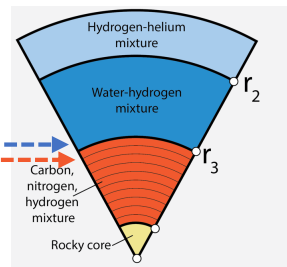
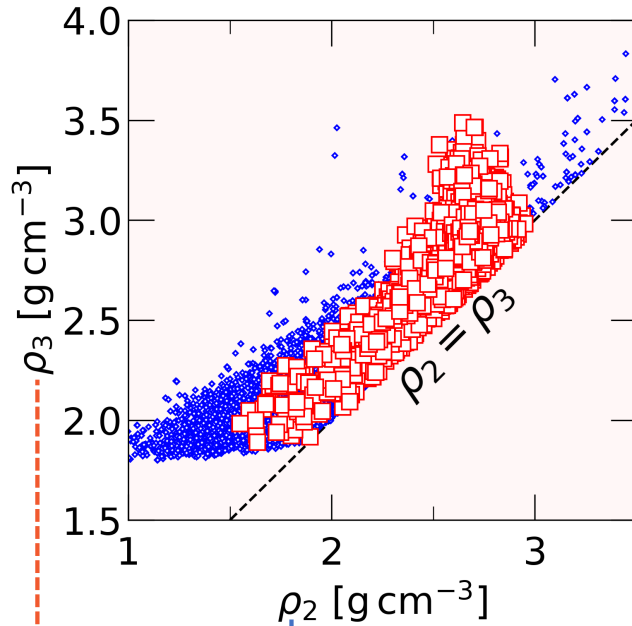
Monte Carlo Ensembles of U+N models that match the Gravity Measurements



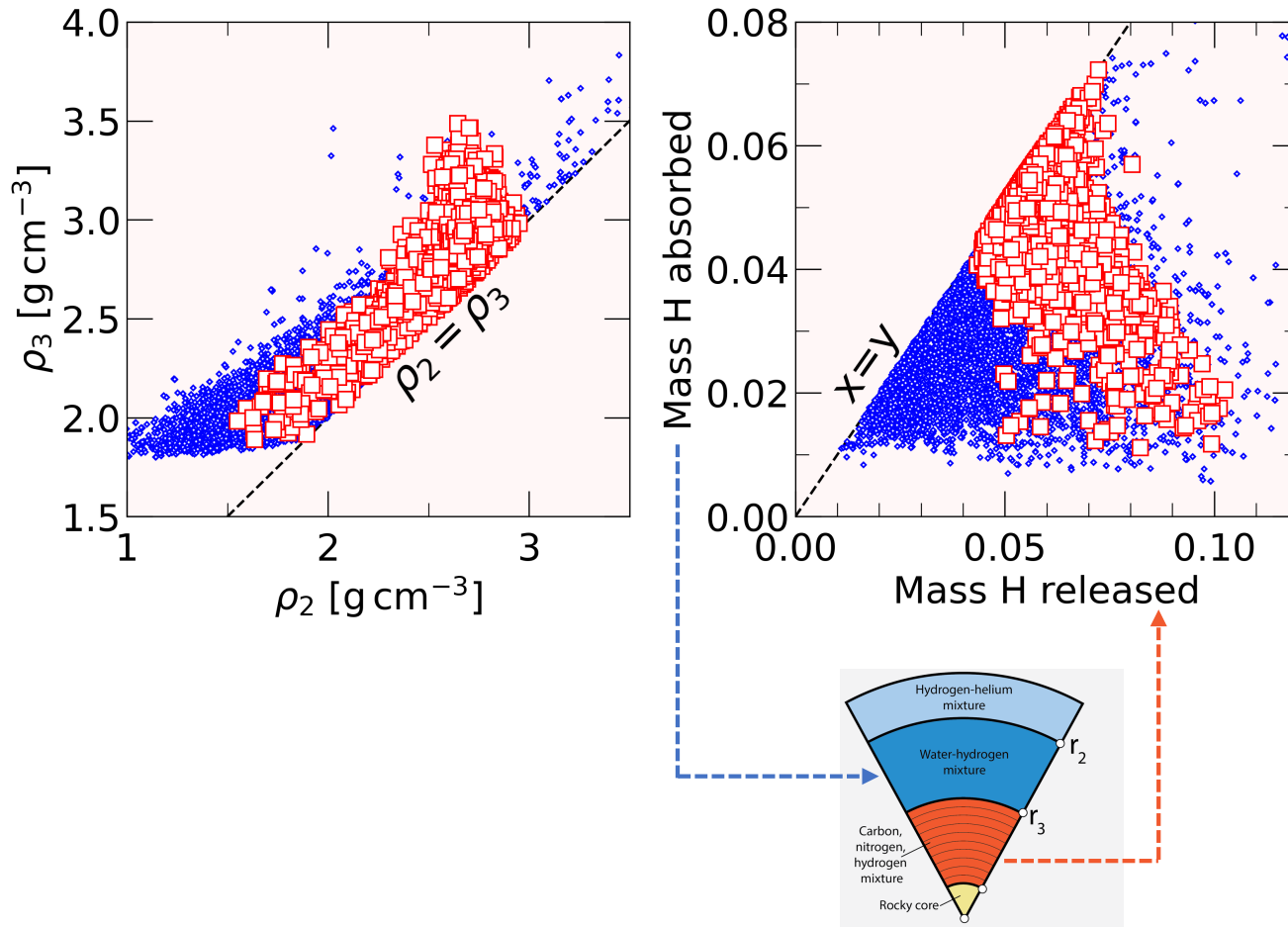
B. Militzer
 Astrophysical J. 953 (2023) 111
 Open QMC source code:
<http://militzer.berkeley.edu/QMC>
[10.5281/zenodo.8038144](https://zenodo.org/record/8038144)

| | Uranus | Neptune |
|----------------------------|--------------------|-----------------|
| Measured $J_2 \times 10^6$ | 3510.99 ± 0.72 | 3529 ± 45 |
| Model $J_2 \times 10^6$ | 3510.99 | 3529.40 |
| Measured $J_4 \times 10^6$ | -33.61 ± 1 | -35.8 ± 2.9 |
| Model $J_4 \times 10^6$ | -33.61 | -35.80 |
| Model $J_6 \times 10^6$ | 0.4859 | 0.5314 |

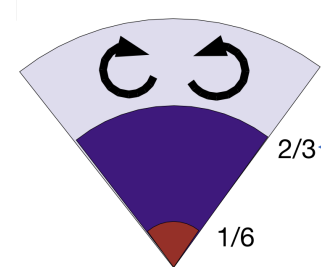
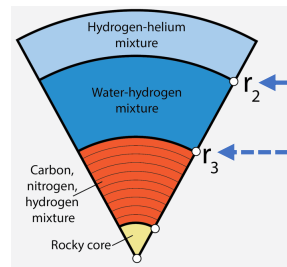
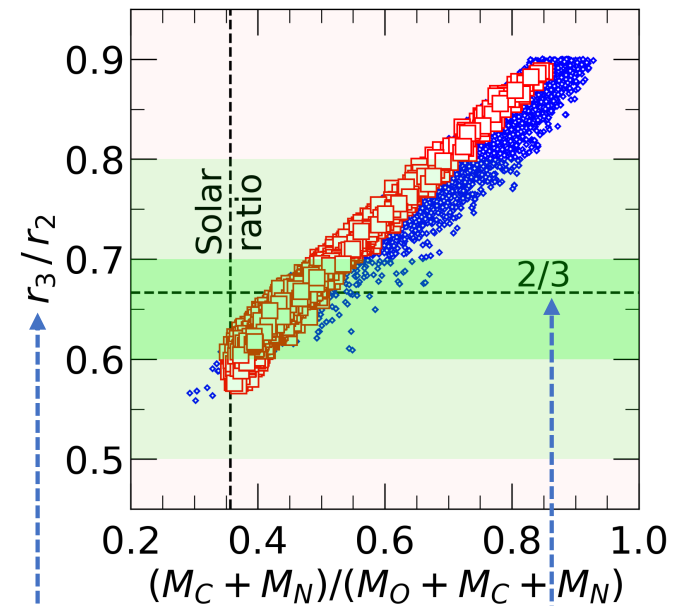
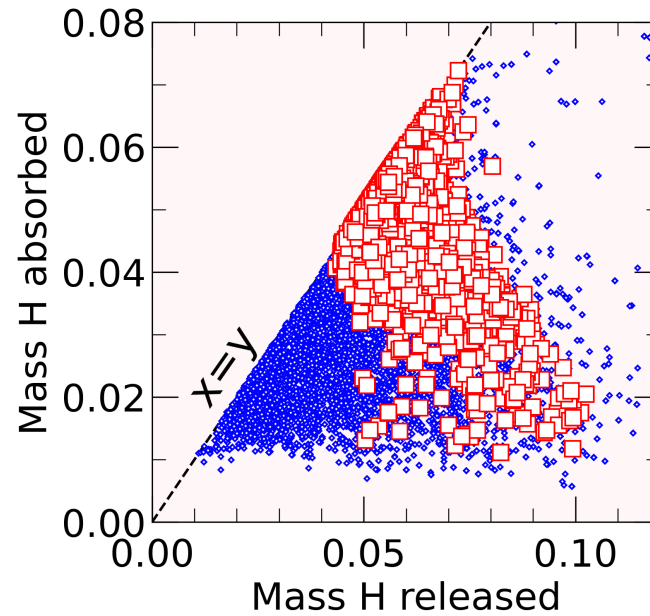
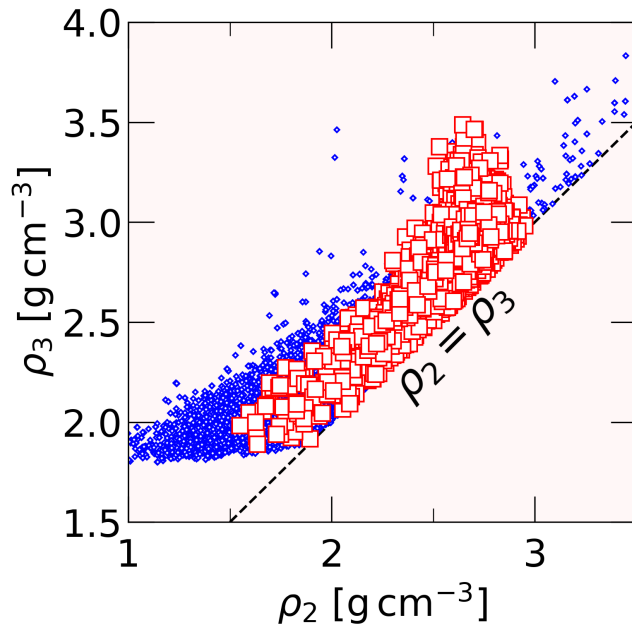
Monte Carlo Ensembles of U+N models that fulfill more constraints (radius 2/3 is compatible with solar)



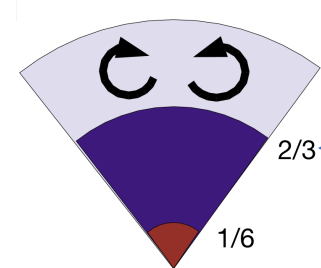
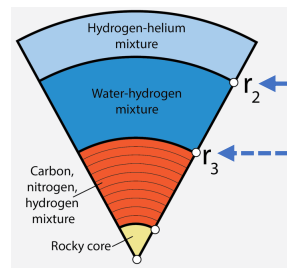
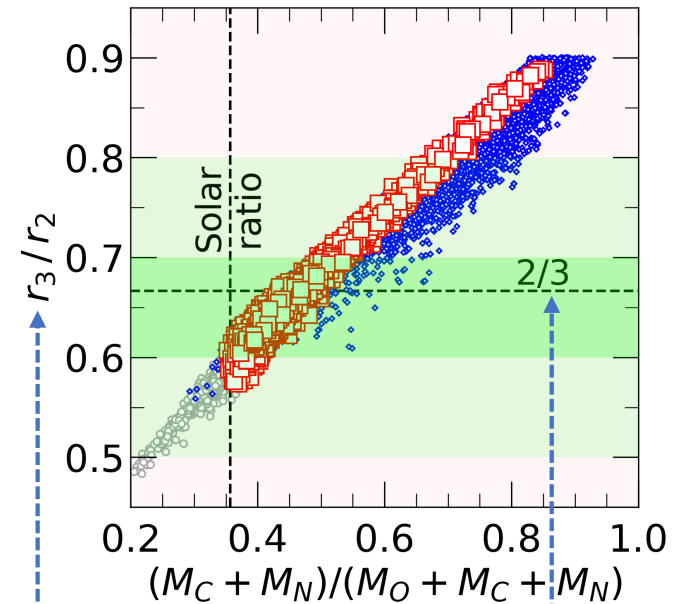
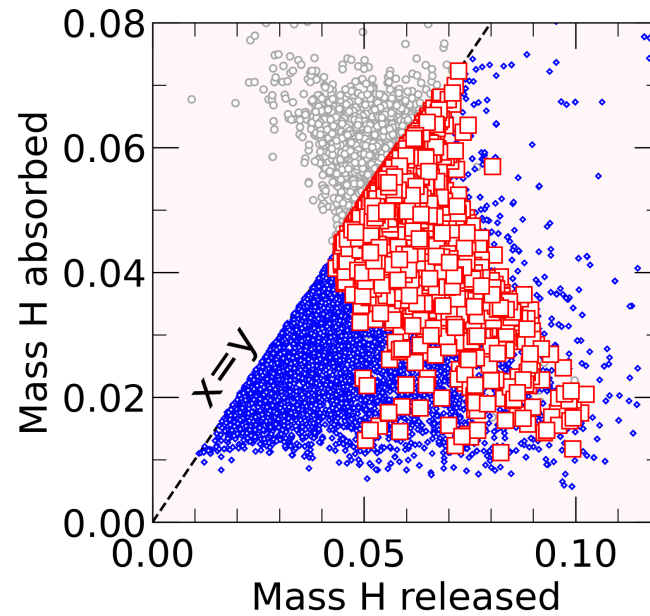
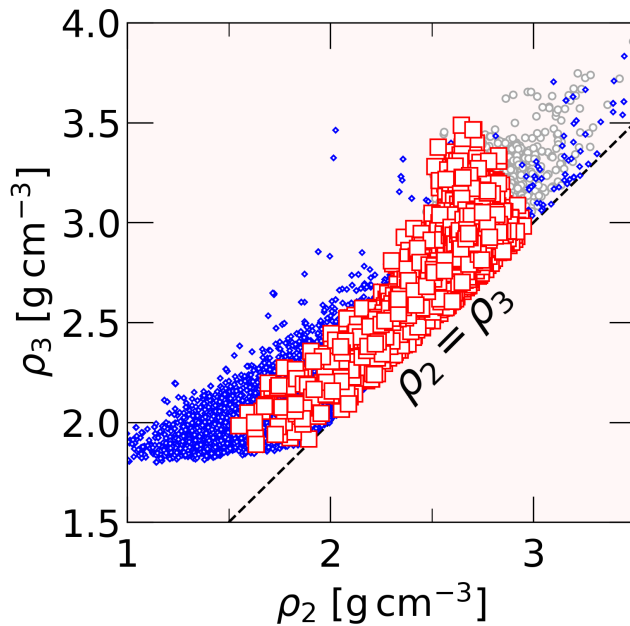
Monte Carlo Ensembles of U+N models that fulfill more constraints (radius 2/3 is compatible with solar)



Monte Carlo Ensembles of U+N models that fulfill more constraints (radius 2/3 is compatible with solar)

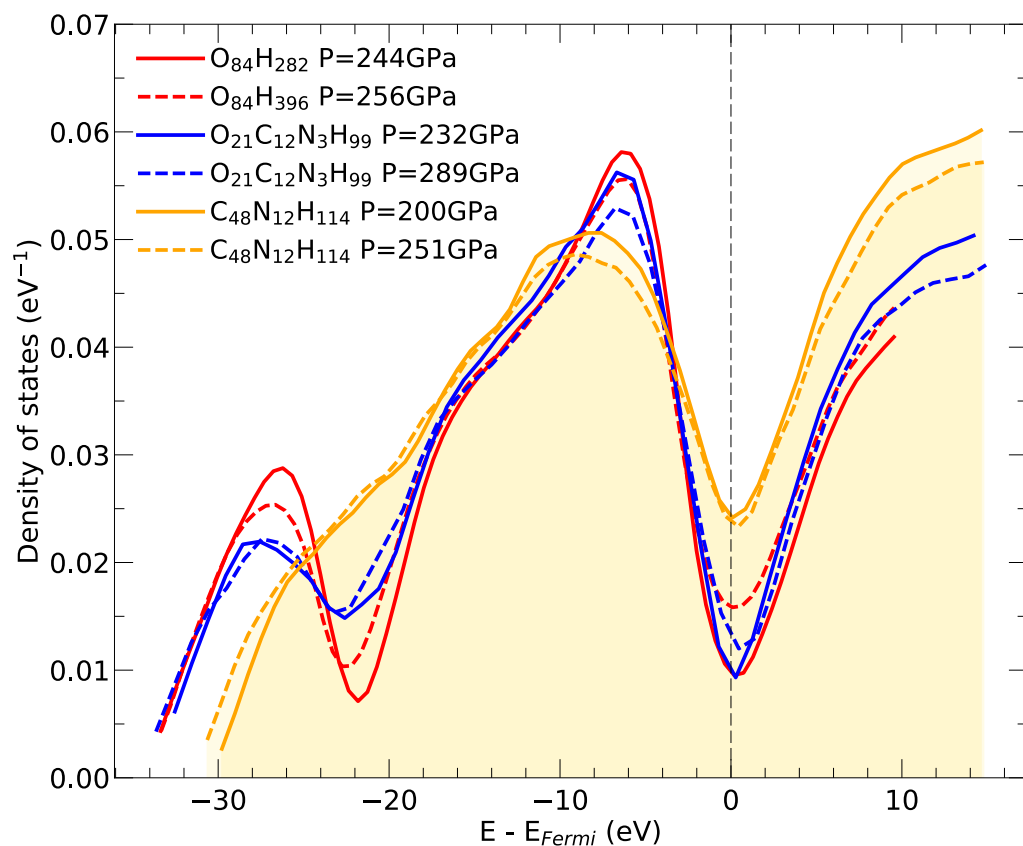


Monte Carlo Ensembles of U+N models that fulfill more constraints (radius 2/3 is compatible with solar)



Are the different materials electrical conductors?

Yes, all three are good “metals”



$\text{O}_{21}\text{N}_3\text{C}_{12}\text{H}_{99}$ mixture

7000 or 11000 S/cm (derived with HSE)

Water-hydrogen mixture

$\text{O}_{84}\text{H}_{228}$ and $\text{O}_{84}\text{H}_{396}$

8000 or 21000 S/cm

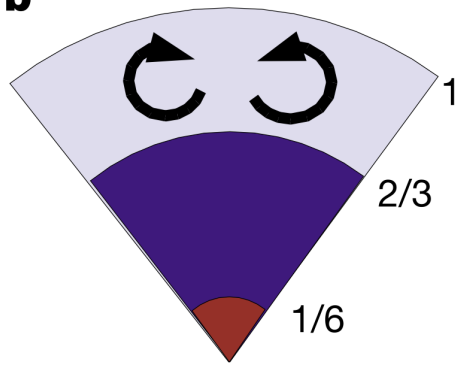
Carbon-nitrogen-hydrogen mixture

$\text{C}_{48}\text{N}_{12}\text{H}_{114}$

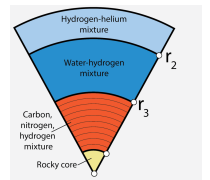
31000 and 35000 S/cm

Our Model for U+N's Interior Structure is Consistent with Stanley & Bloxham's Predictions

b



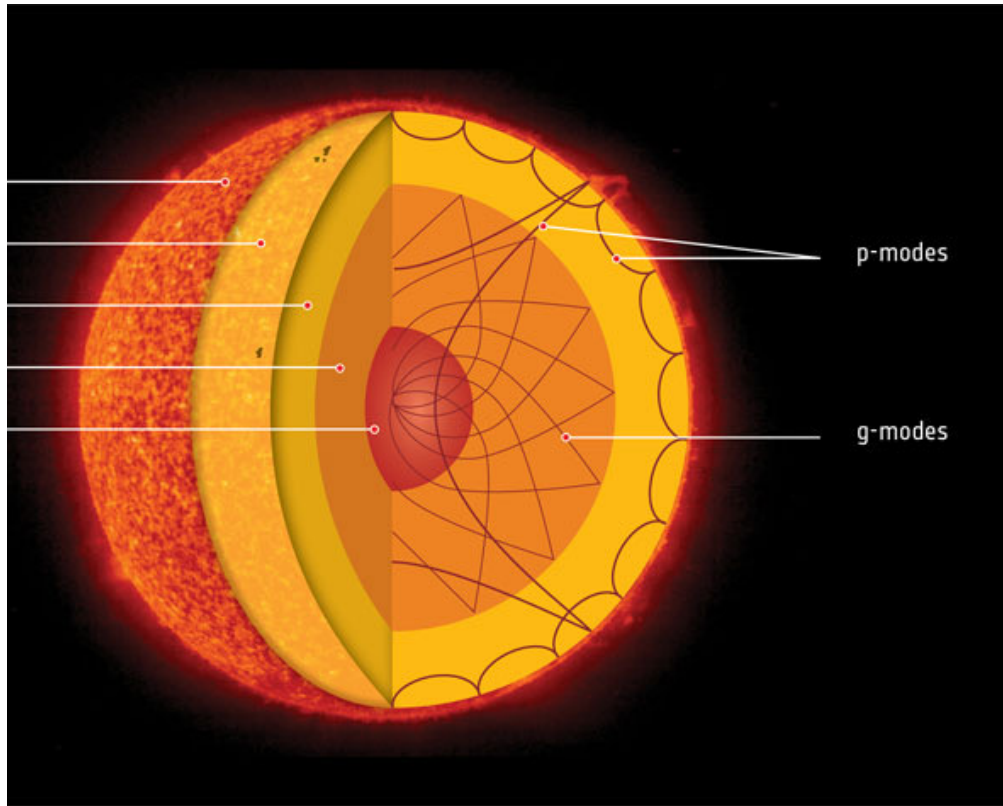
- ✓ ☒ U+N's magnetic fields are primarily generated in a thin outer layer
- ✓ ☒ This layer a homogeneous, electrically conducting fluid
- ✓ ☒ The inner-outer radius boundary is approximated at $2/3$.
- ✓ ☒ The inner layer is non-convecting, electrically conducting fluid
- ✓ ☒ They prefer a stably stratified electrically conducting fluid but a non-convecting, conducting solid might also work.



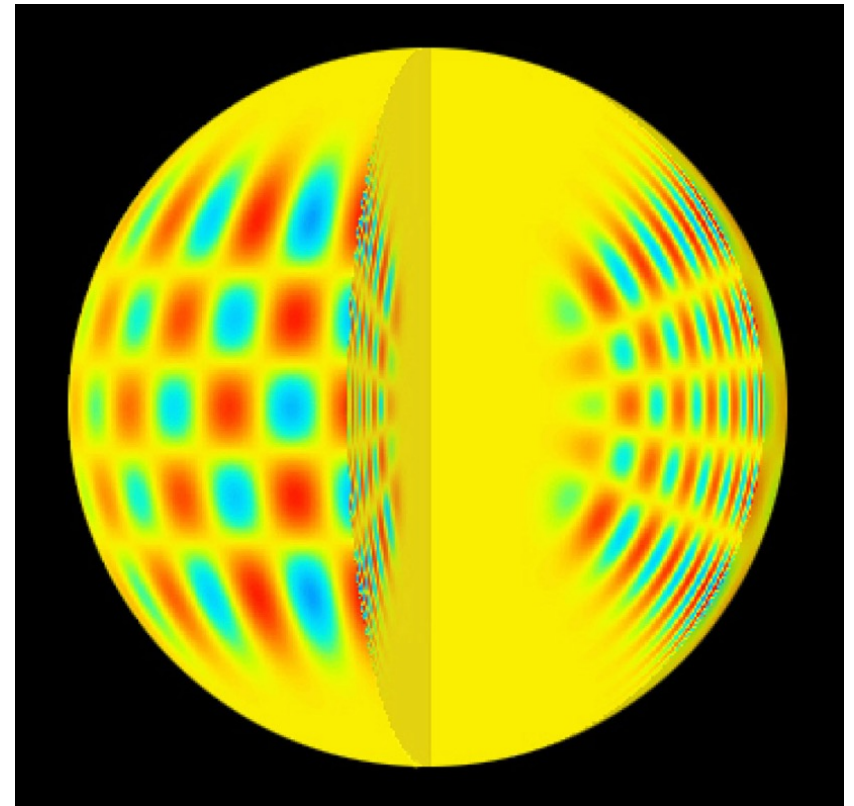
Open questions:

- 1) What is the composition of the upper layer? **A conducting, fluid mixture of H_2O and H_2**
- 2) What is the composition of the lower layer? **Conducting carbon-nitrogen-hydrogen fluid**
- 3) Why is the lower layer not convective? **The amount of hydrogen varies with depth. Leading to density stratification that prevents convection.**

Helioseismology

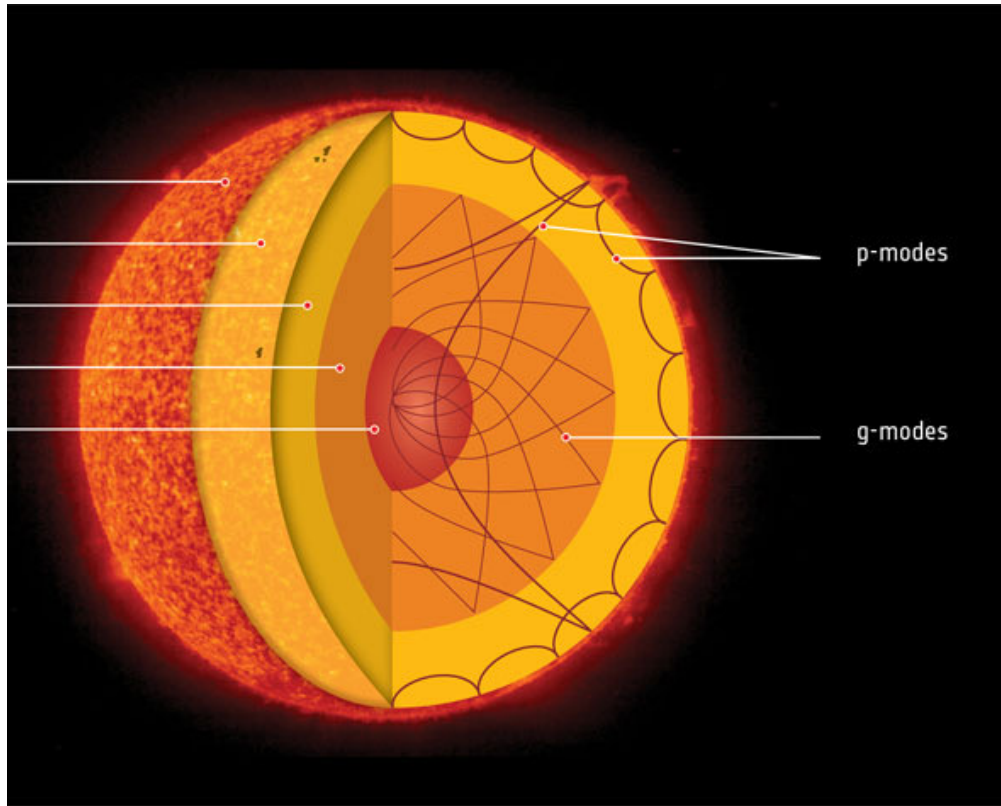


NASA SOHO

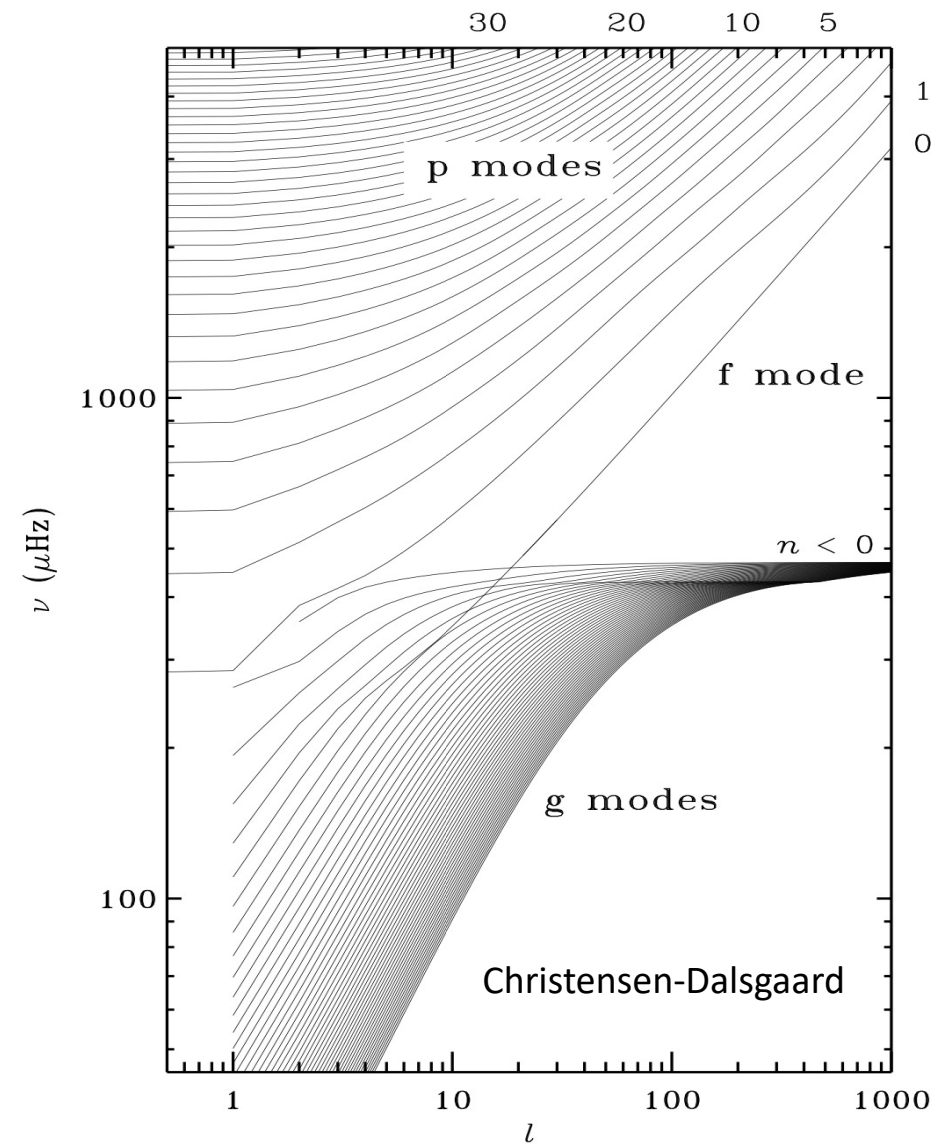


P modes

Helioseismology



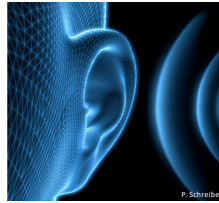
NASA SOHO



Helioseismology

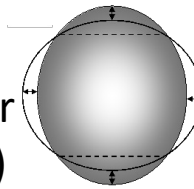
P modes: “Pressure” waves

- Primary restoring force is pressure
- High-frequency limit: acoustic waves



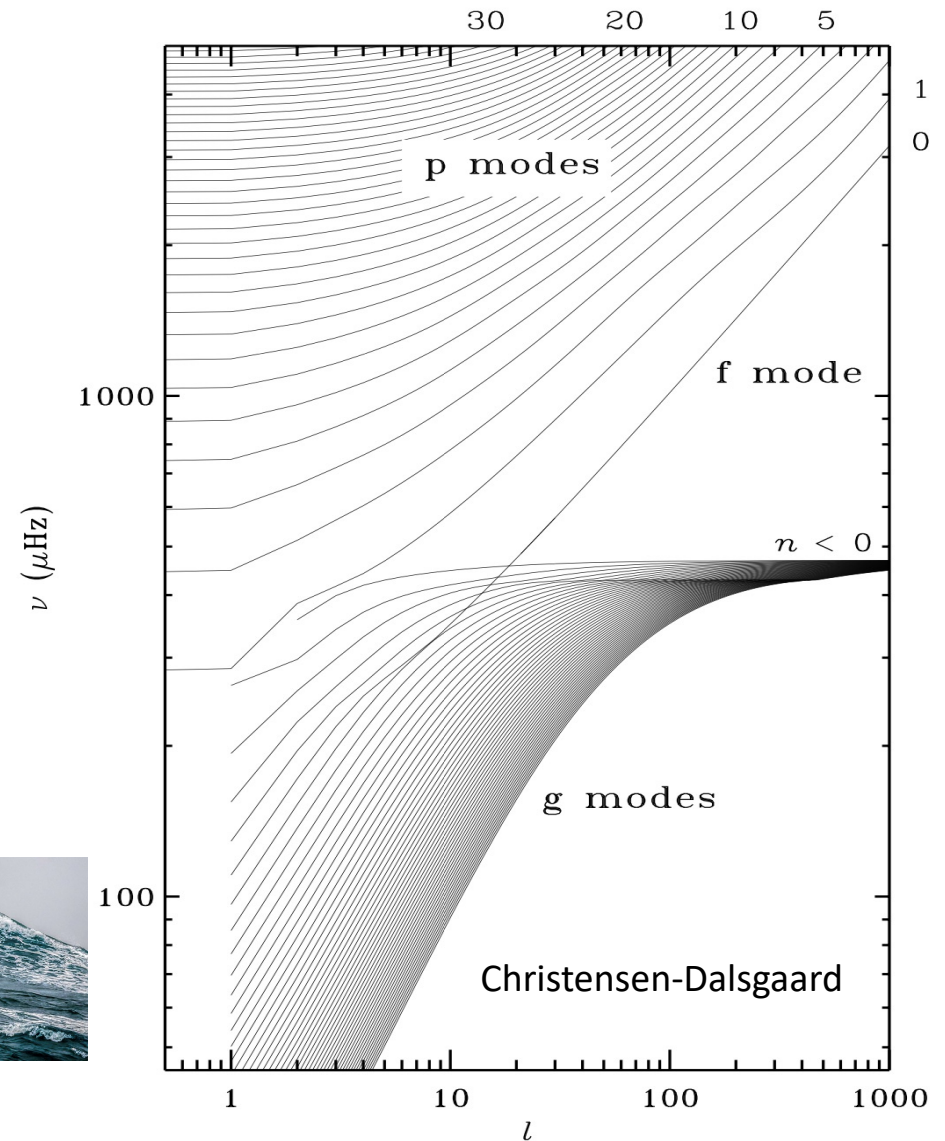
F modes: “Fundamental” modes

- Are the limit of p modes as radial order n goes to zero (long-wavelength limit)
- Also known as surface gravity wave, no nodes in interior. Deforms like a soccer ball
- No compression involved.

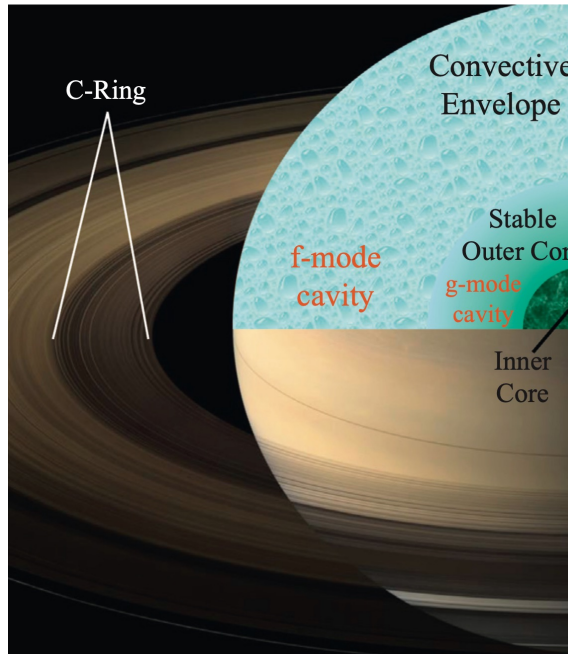


G Modes: “Gravity” waves

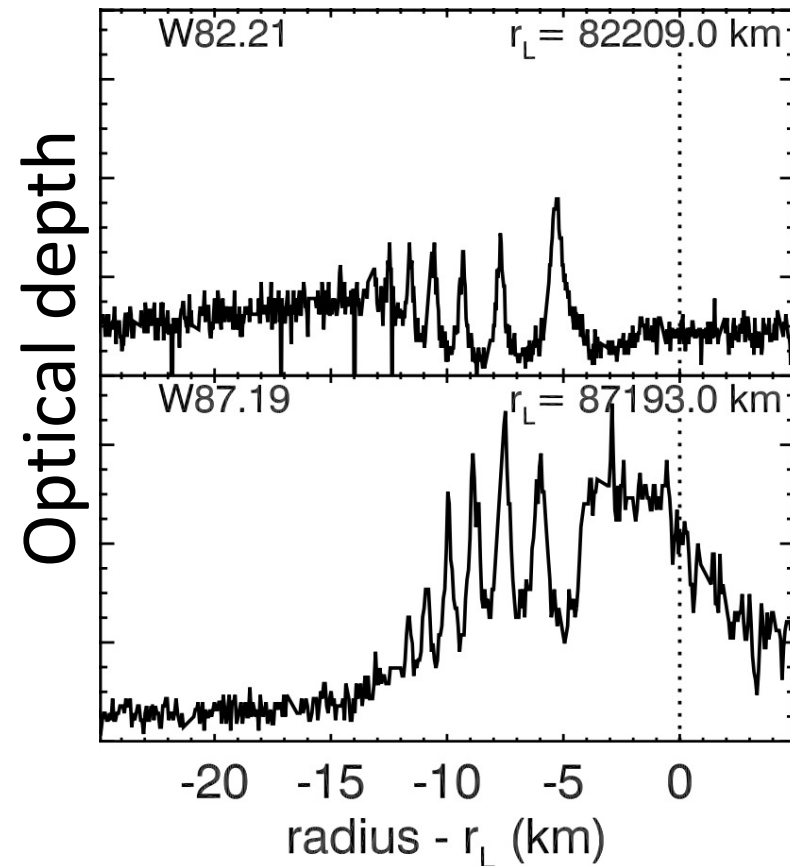
- Low frequency waves
- Primary restoring force is buoyancy
- Requires stable stratification, no convection



Saturn's Rings are a Seismometer, Spiral Density Waves



Fuller (2014)



Hedman & Nicolson (2013)

Take-Away Points

1. Ab initio simulations predict a **O-C-N-H mixture to phase separated** into a O-H and a C-N-H fluid at high pressure
2. Constructed planet model for Uranus and Neptune. Their icy mantles of have **two layers**: an upper **H₂O-H₂ layer** and lower **stably stratified C-N-H layer**
3. Under these assumptions we can match the **gravity and magnetic field** measurements
4. A Uranus spacecraft should bring a **Doppler imager**.